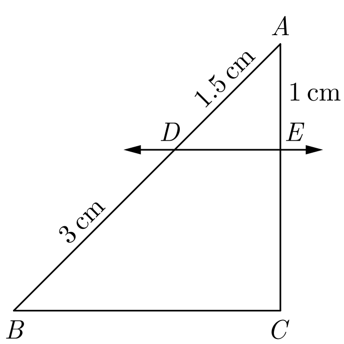
CHAPTER 6

TRIANGLES

ONE MARK QUESTIONS

MULTIPLE CHOICE QUESTIONS

1. In the given figure, $DE \parallel BC$. The value of EC is



(a) 1.5 cm (b) 3 cm

(c) 2 cm (d) 1 cm

Ans :

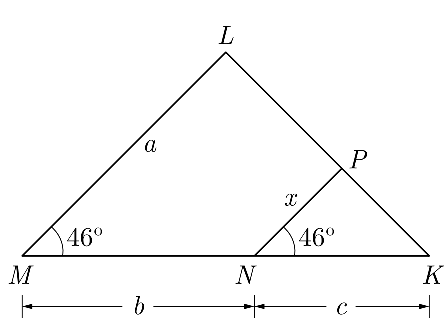
Since, $DE \parallel BC$

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{1.5}{3} = \frac{1}{EC} \Rightarrow EC = 2 \text{ cm}$$

Thus (c) is correct option.

2. In the given figure, x is



(a) $\frac{ab}{a+b}$ (b) $\frac{ac}{b+c}$

(c) $\frac{bc}{b+c}$ (d) $\frac{ac}{a+c}$

Ans :

In ΔKPN and ΔKLM, $\angle K$ is common and we have

$$\angle KNP = \angle KML = 46^\circ$$

Thus by $A-A$ criterion of similarity,

$$\Delta KNP \sim \Delta KML$$

Thus $$\frac{KN}{KM} = \frac{NP}{ML}$$

$$\frac{c}{b+c} = \frac{x}{a} \Rightarrow x = \frac{ac}{b+c}$$

Thus (b) is correct option.

3. ΔABC is an equilateral triangle with each side of length $2p$. If $AD \perp BC$ then the value of AD is

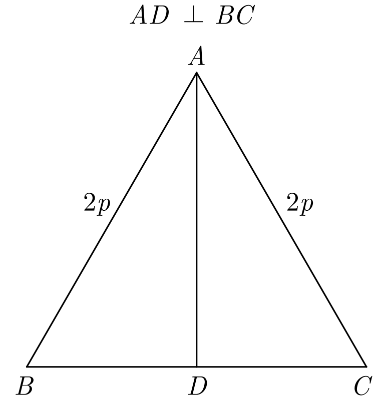
(a) $\sqrt{3}$ (b) $\sqrt{3}\,p$

(c) $2p$ (d) $4p$

Ans :

We have $AB = BC = CA = 2p$

and $AD \perp BC$



In ΔADB, $AB^2 = AD^2 + BD^2$

$$(2p)^2 = AD^2 + p^2$$

$$AD^2 = \sqrt{3}\,p$$

Thus (b) is correct option.

4. Which of the following statement is false?

(a) All isosceles triangles are similar.

(b) All quadrilateral are similar.

(c) All circles are similar.

(d) None of the above

Ans :

Isosceles triangle is a triangle in which two side of equal length. Thus two isosceles triangles may not be similar. Hence statement given in option (a) is false.

Thus (a) is correct option.

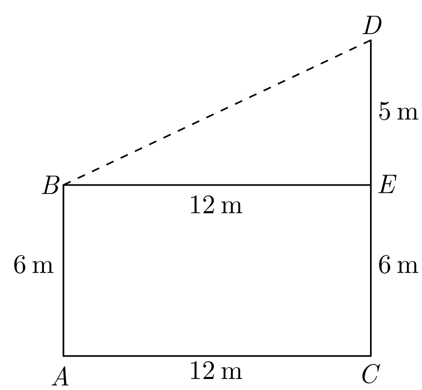
5. Two poles of height 6 m and 11 m stand vertically upright on a plane ground. If the distance between their foot is 12 m, then distance between their tops is

(a) 12 m (b) 14 m

(c) 13 m (d) 11 m

Ans :

Let AB and CD be the vertical poles as shown below.



We have $\quad AB = 6\text{ m},\ CD = 11\text{ m}$

and $\quad AC = 12\text{ m}$

$$DE = CD - CE$$

$$= (11-6)\text{ m} = 5\text{ m}$$

In right angled, ΔBED,

$$BD^2 = BE^2 + DE^2 = 12^2 + 5^2 = 169$$

$$BD = \sqrt{169}\text{ m} = 13\text{ m}$$

Hence, distance between their tops is 13 m.

Thus (c) is correct option.

6. In a right angled ΔABC right angled at B, if P and Q are points on the sides AB and BC respectively, then

(a) $AQ^2 + CP^2 = 2(AC^2 + PQ^2)$

(b) $2(AQ^2 + CP^2) = AC^2 + PQ^2$

(c) $AQ^2 + CP^2 = AC^2 + PQ^2$

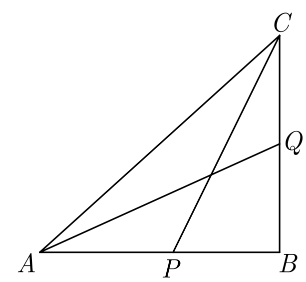
(d) $AQ + CP = \frac{1}{2}(AC + PQ)$

Ans :

In right angled ΔABQ and ΔCPB,

$$CP^2 = CB^2 + BP^2$$

and $\quad AQ^2 = AB^2 + BQ^2$



$$CP^2 + AQ^2 = CB^2 + BP^2 + AB^2 + BQ^2$$

$$= CB^2 + AB^2 + BP^2 + BQ^2$$

$$= AC^2 + PQ^2$$

Thus (c) is correct option.

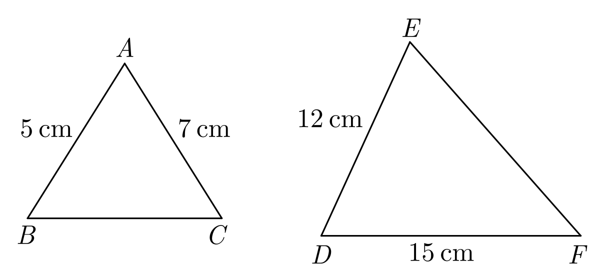
7. It is given that, $\Delta ABC \sim \Delta EDF$ such that $AB = 5$ cm, $AC = 7$ cm, $DF = 15$ cm and $DE = 12$ cm then the sum of the remaining sides of the triangles is

(a) 23.05 cm (b) 16.8 cm

(c) 6.25 cm (d) 24 cm

Ans :

We have $\quad \Delta ABC \sim \Delta EDF$



Now $\quad \frac{5}{12} = \frac{7}{EF} = \frac{BC}{15}$

Taking first and second ratios, we get

$$\frac{5}{12} = \frac{7}{EF} \Rightarrow EF = \frac{7 \times 12}{5}$$

$$= 16.8 \text{ cm}$$

Taking first and third ratios, we get

$$\frac{5}{12} = \frac{BC}{15} \Rightarrow BC = \frac{5 \times 15}{12}$$

$$= 6.25 \text{ cm}$$

Now, sum of the remaining sides of triangle,

$$EF + BC = 16.8 + 6.25 = 23.05 \text{ cm}$$

Thus (a) is correct option.

8. The area of a right angled triangle is 40 sq cm and its perimeter is 40 cm. The length of its hypotenuse is

(a) 16 cm (b) 18 cm

(c) 17 cm (d) data insufficient

Ans : (b) 18 cm

Let c be the hypotenuse of the triangle, a and b be other sides.

Now $c = \sqrt{a^2 + b^2}$

We have, $a + b + c = 40$ and $\frac{1}{2}ab = 40 \Rightarrow ab = 80$

$$c = 40 - (a + b) \text{ and } ab = 80$$

Squaring $c = 40 - (a + b)$ we have

$$c^2 = [40 - (a + b)]^2$$

$$a^2 + b^2 = 1600 - 2 \times 40(a + b) + (a + b)^2$$

$$a^2 + b^2 = 1600 - 2 \times 40(a + b) + a^2 + 2ab + b^2$$

$$0 = 1600 - 2 \times 40(a + b) + 2 \times 80$$

$$0 = 20 - (a + b) + 2$$

$$a + b = 22$$

$$c = 40 - (a + b) = 40 - 22 = 18 \text{ cm}$$

Thus (b) is correct option.

9. **Assertion :** In the $\triangle ABC$, $AB = 24$ cm, $BC = 10$ cm and $AC = 26$ cm, then $\triangle ABC$ is a right angle triangle.

Reason : If in two triangles, their corresponding angles are equal, then the triangles are similar.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans :

We have,

$$AB^2 + BC^2 = (24)^2 + (10)^2$$

$$= 576 + 100 = 676 = AC^2$$

Thus $AB^2 + BC^2 = AC^2$ and ABC is a right angled triangle.

Also, two triangle are similar if their corresponding angles are equal.

Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

Thus (b) is correct option.

Fill in the Blank Questions

10. A line drawn through the mid-point of one side of a triangle parallel to another side bisects the .... side.

Ans :

third

11. .... theorem states that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Ans :

Pythagoras

12. Line joining the mid-points of any two sides of a triangle is .... to the third side.

Ans :

parallel

13. All squares are ....

Ans :

similar

14. Two triangles are said to be .... if corresponding angles of two triangles are equal.

Ans :

equiangular

15. All similar figures need not be ....

Ans :

congruent

16. All circles are ....

Ans :

similar

17. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the .... side.

Ans :

third

18. If a line divides any two sides of a triangle in the same ratio, then the line is .... to the third side.

Ans :

parallel

19. All congruent figures are similar but the similar figures need .... be congruent.

Ans :

not

20. Two figures are said to be .... if they have same shape but not necessarily the same size.

Ans :

similar

21. .... theorem states that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Ans :

Basic proportionality

22. All .... triangles are similar.

Ans :

equilateral

23. Two figures having the same shape and size are said to be ....

Ans :

congruent

24. Two triangles are similar if their corresponding sides are ....

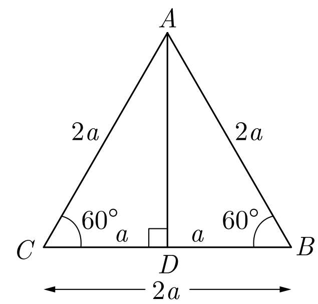
Ans :

in the same ratio.

25. ΔABC is an equilateral triangle of side $2a$, then length of one of its altitude is ....

Ans : [Board 2020 Delhi Standard]

ΔABC is an equilateral triangle as shown below, in which $AD \perp BC$.



Using Pythagoras theorem we have

$$AB^2 = (AD)^2 + (BD)^2$$

$$(2a)^2 = (AD)^2 + (a)^2$$

$$4a^2 - a^2 = (AD)^2$$

$$(AD)^2 = 3a^2$$

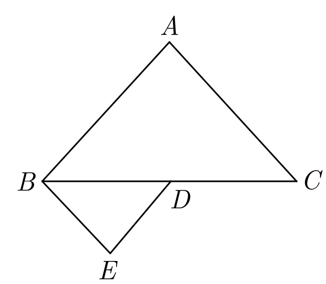
$$AD = a\sqrt{3}$$

Hence, the length of attitude is $a\sqrt{3}$.

26. ΔABC and ΔBDE are two equilateral triangle such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is ....

Ans : [Board 2020 Delhi Standard]

From the given information we have drawn the figure as below.



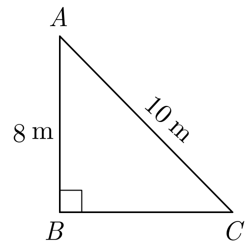
$$\frac{ar(\Delta ABC)}{ar(\Delta BDE)} = \frac{\frac{\sqrt{3}}{4}(BC)^2}{\frac{\sqrt{3}}{4}(BD)^2} = \frac{(BC)^2}{(\frac{1}{2}BC)^2}$$

$$= \frac{4BC^2}{BC^2} = \frac{4}{1} = 4:1$$

27. A ladder 10 m long reaches a window 8 m above the ground. The distance of the foot of the ladder from the base of the wall is .... m.

Ans : [Board 2020 Delhi Standard]

Let AB be the height of the window above the ground and BC be a ladder.



Here, $AB = 8$ m

and $AC = 10$ m

In right angled triangle ABC,

$$AC^2 = AB^2 + BC^2$$

$$10^2 = 8^2 + BC^2$$

$$BC^2 = 100 - 64 = 36$$

$$BC = 6 \text{ m}$$

28. In ΔABC, $AB = 6\sqrt{3}$ cm, $AC = 12$ cm and $BC = 6$ cm, then $\angle B =$ ....

Ans : [Board 2020 OD Standard]

We have $AB = 6\sqrt{3}$ cm,

$AC = 12$ cm and

$BC = 6$ cm

Now $AB^2 = 36 \times 3 = 108$

$$AC^2 = 144$$

and $BC^2 = 36$

In can be easily observed that above values satisfy Pythagoras theorem,

$$AB^2 + BC^2 = AC^2$$

$$108 + 36 = 144 \text{ cm}$$

Thus $\angle B = 90^\circ$

29. The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of the first triangle is 9 cm, then the corresponding side of second triangle is ....

Ans : [Board 2020 Delhi Basic]

Ratio of the perimeter of two similar triangles is equal to the ratio of corresponding sides.

Thus $\frac{25}{15} = \frac{9}{\text{side}}$

$$\text{side} = \frac{9 \times 15}{25} = 5.4 \text{ cm}$$

Very Short Answer Questions

30. ΔABC is isosceles with $AC = BC$. If $AB^2 = 2AC^2$, then find the measure of $\angle C$.

Ans : [Board 2020 Delhi Basic]

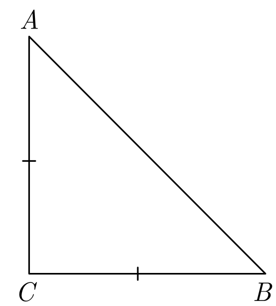
We have $AB^2 = 2AC^2$

$$AB^2 = AC^2 + AC^2$$

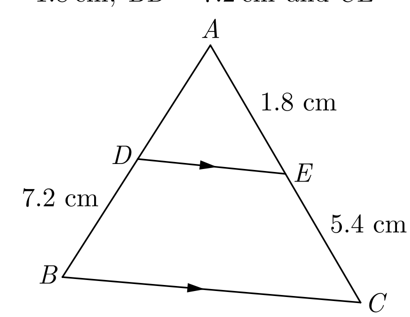
$$AB^2 = BC^2 + AC^2$$

$(BC = AC)$

It satisfies the Pythagoras theorem. Thus according to converse of Pythagoras theorem, ΔABC is a right angle triangle and $\angle C = 90^\circ$.



31. In Figure, $DE || BC$. Find the length of side AD, given that $AE = 1.8$ cm, $BD = 7.2$ cm and $CE = 5.4$ cm.



Ans : [Board 2019 OD]

Since $DE || BC$ we have

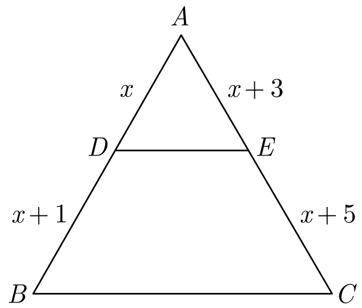
$$\frac{AD}{DB} = \frac{AE}{EC}$$

Substituting the values, we get

$$\frac{AD}{7.2} = \frac{1.8}{5.4}$$

$$AD = \frac{1.8 \times 7.2}{5.4} = \frac{12.96}{5.4} = 2.4 \text{ cm}$$

32. In $\Delta ABC, DE || BC$, find the value of x.



Ans : [Board Term-1 2016]

In the given figure $DE || BC$, thus

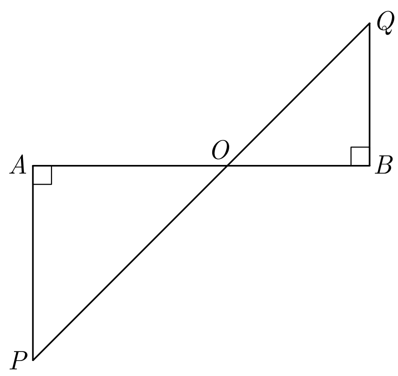
$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{x+1} = \frac{x+3}{x+5}$$

$$x^2 + 5x = x^2 + 4x + 3$$

$$x = 3$$

33. In the given figure, if $\angle A = 90^0, \angle B = 90^0$, $OB = 4.5$ cm $OA = 6$ cm and $AP = 4$ cm then find QB.



Ans : [Board Term-1, 2015]

In ΔPAO and ΔQBO we have

$$\angle A = \angle B = 90^0$$

Vertically opposite angle,

$$\angle POA = \angle QOB$$

Thus $\Delta PAO \sim \Delta QBO$

$$\frac{OA}{OB} = \frac{PA}{QB}$$

$$\frac{6}{4.5} = \frac{4}{QB}$$

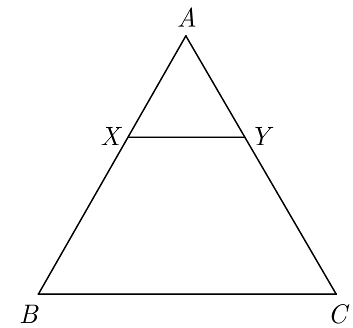
$$QB = \frac{4 \times 4.5}{6} = 3 \text{ cm}$$

Thus $QB = 3$ cm

34. In ΔABC, if X and Y are points on AB and AC respectively such that $\frac{AX}{XB} = \frac{3}{4}$, $AY = 5$ and $YC = 9$, then state whether XY and BC parallel or not.

Ans : [Board Term-1 2016, 2015]

As per question we have drawn figure given below.



In this figure we have

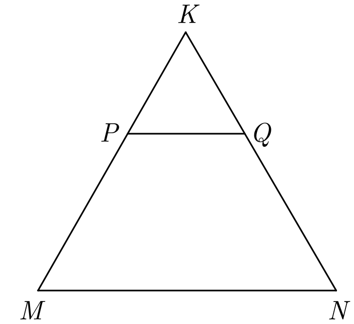
$$\frac{AX}{XB} = \frac{3}{4}, AY = 5 \text{ and } YC = 9$$

Now $$\frac{AX}{XB} = \frac{3}{4} \text{ and } \frac{AY}{YC} = \frac{5}{9}$$

Since $$\frac{AX}{XB} \neq \frac{AY}{YC}$$

Hence XY is not parallel to BC.

35. In the figure, PQ is parallel to MN. If $\frac{KP}{PM} = \frac{4}{13}$ and $KN = 20.4$ cm then find KQ.



Ans :

In the given figure $PQ || MN$, thus

$$\frac{KP}{PM} = \frac{KQ}{QN} \quad \text{(By BPT)}$$

$$\frac{KP}{PM} = \frac{KQ}{KN - KQ}$$

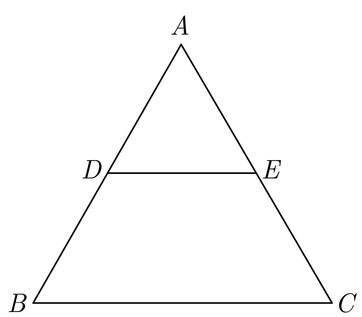
$$\frac{4}{13} = \frac{KQ}{20.4 - KQ}$$

$$4 \times 20.4 - 4KQ = 13KQ$$

$$17KQ = 4 \times 20.4$$

$$KQ = \frac{20.4 \times 4}{17} = 4.8 \text{ cm}$$

36. In given figure $DE || BC$. If $AD = 3c, DB = 4c$ cm and $AE = 6$ cm then find EC.



Ans : [Board Term-1 2016]

In the given figure $DE \parallel BC$, thus

$$\frac{AD}{BD} = \frac{AE}{EC}$$

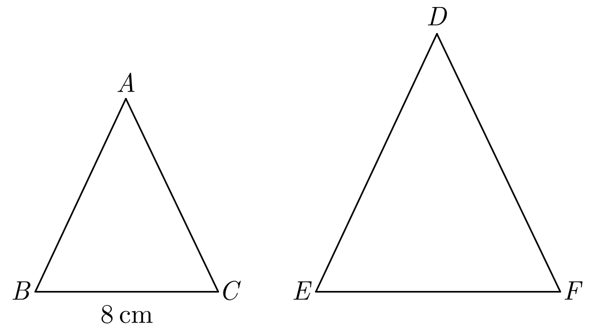
$$\frac{3}{4} = \frac{6}{EC}$$

$$EC = 8 \text{ cm}$$

37. If triangle ABC is similar to triangle DEF such that $2AB = DE$ and $BC = 8$ cm then find EF.

Ans :

As per given condition we have drawn the figure below.



Here we have $2AB = DE$ and $BC = 8$ cm

Since $\Delta ABC \sim \Delta DEF$, we have

$$\frac{AB}{BC} = \frac{DE}{EF}$$

$$\frac{AB}{8} = \frac{2AB}{EF}$$

$$EF = 2 \times 8 = 16 \text{ cm}$$

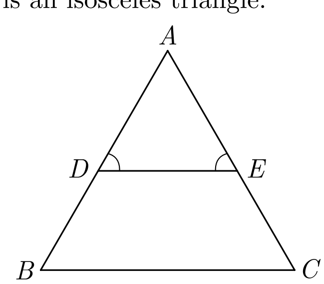
38. Are two triangles with equal corresponding sides always similar?

Ans : [Board Term-1 2015]

Yes, Two triangles having equal corresponding sides are are congruent and all congruent Δs have equal angles, hence they are similar too.

TWO MARKS QUESTIONS

39. In Figure $\angle D = \angle E$ and $\frac{AD}{DB} = \frac{AE}{EC}$, prove that ΔBAC is an isosceles triangle.



Ans : [Board 2020 Delhi Standard]

We have, $\angle D = \angle E$

and $\frac{AD}{DB} = \frac{AE}{EC}$

By converse of BPT, $DE \parallel BC$

Due to corresponding angles we have

$$\angle ADE = \angle ABC \text{ and}$$

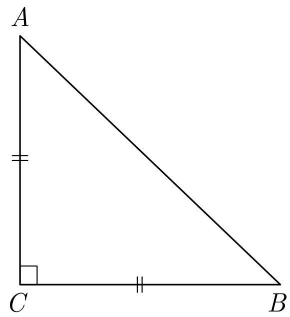
$$\angle AED = \angle ACB$$

Given $\angle ADE = \angle AED$

Thus $\angle ABC = \angle ACB$

Therefore BAC is an isosceles triangle.

40. In Figure, ABC is an isosceles triangle right angled at C with $AC = 4$ cm, Find the length of AB.



Ans : [Board 2019 OD]

Since ABC is an isosceles triangle right angled at C,

$$AC = BC = 4\text{ cm}$$

$$\angle C = 90^\circ$$

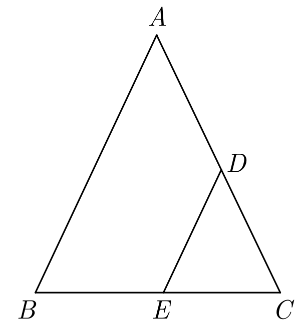
Using Pythagoras theorem in ΔABC we have,

$$AB^2 = BC^2 + AC^2$$

$$= 4^2 + 4^2 = 16 + 16 = 32$$

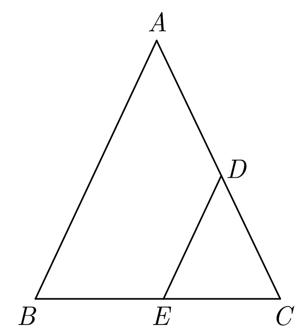
$$AB = 4\sqrt{2}\text{ cm.}$$

41. In the figure of ΔABC, the points D and E are on the sides CA, CB respectively such that $DE || AB$, $AD = 2x, DC = x + 3, BE = 2x - 1$ and $CE = x$. Then, find x.



OR

In the figure of ΔABC, $DE || AB$. If $AD = 2x$, $DC = x + 3, BE = 2x - 1$ and $CE = x$, then find the value of x.



Ans : [Board Term-1 2015, 2016]

We have $\dfrac{CD}{AD} = \dfrac{CE}{BE}$

$$\frac{x+3}{2x} = \frac{x}{2x-1}$$

$$5x = 3 \text{ or, } x = \frac{3}{5}$$

Alternative Method :

In ABC, $DE || AB$, thus

$$\frac{CD}{CA} = \frac{CE}{CB}$$

$$\frac{CD}{CA - CD} = \frac{CE}{CB - CE}$$

$$\frac{CD}{AD} = \frac{CE}{BE}$$

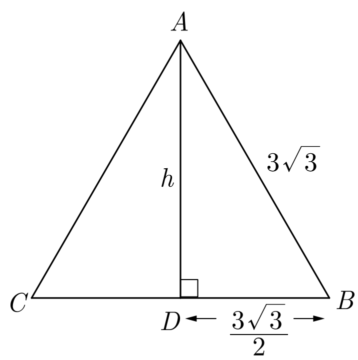
$$\frac{x+3}{2x} = \frac{x}{2x-1}$$

$$5x = 3 \text{ or, } x = \frac{3}{5}$$

42. In an equilateral triangle of side $3\sqrt{3}$ cm find the length of the altitude.

Ans : [Board Term-1 2016]

Let ΔABC be an equilateral triangle of side $3\sqrt{3}$ cm and AD is altitude which is also a perpendicular bisector of side BC. This is shown in figure given below.



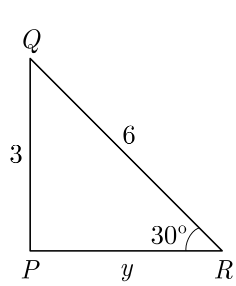
Now $$(3\sqrt{3})^2 = h^2 + \left(\frac{3\sqrt{3}}{2}\right)^2$$

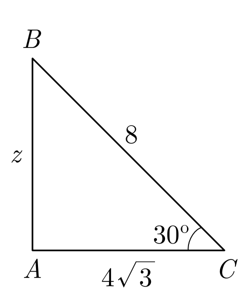
$$27 = h^2 + \frac{27}{4}$$

$$h^2 = 27 - \frac{27}{4} = \frac{81}{4}$$

$$h = \frac{9}{2} = 4.5 \text{ cm}$$

43. In the given figure, $\Delta ABC \sim \Delta PQR$. Find the value of $y + z$.





Ans : [Board Term-1 2010]

In the given figure $\Delta ABC \sim \Delta PQR$,

Thus $$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

$$\frac{z}{3} = \frac{8}{6} = \frac{4\sqrt{3}}{y}$$

$$\frac{z}{3} = \frac{8}{6} \text{ and } \frac{8}{6} = \frac{4\sqrt{3}}{y}$$

$$z = \frac{8 \times 3}{6} \text{ and } y = \frac{4\sqrt{3} \times 6}{8}$$

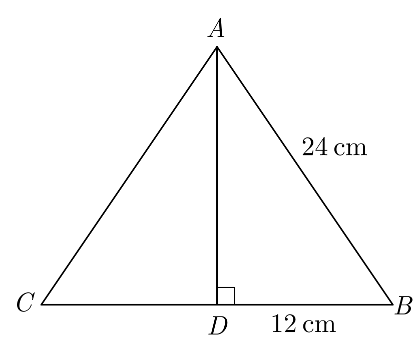
$$z = 4 \text{ and } y = 3\sqrt{3}$$

Thus $$y + z = 3\sqrt{3} + 4$$

44. In an equilateral triangle of side 24 cm, find the length of the altitude.

Ans : [Board Term-1 2015]

Let ΔABC be an equilateral triangle of side 24 cm and AD is altitude which is also a perpendicular bisector of side BC. This is shown in figure given below.



Now $$BD = \frac{BC}{2} = \frac{24}{2} = 12 \text{ cm}$$

$$AB = 24 \text{ cm}$$

$$AD = \sqrt{AB^2 - BD^2}$$

$$= \sqrt{(24)^2 - (12)^2}$$

$$= \sqrt{576 - 144}$$

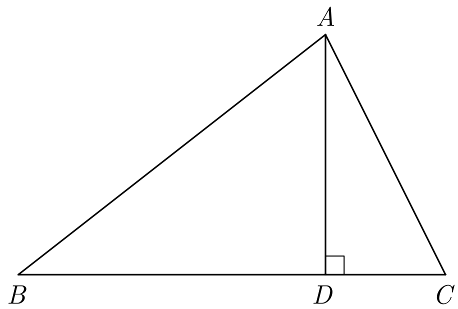
$$= \sqrt{432} = 12\sqrt{3}$$

Thus $AD = 12\sqrt{3}$ cm.

45. In $\Delta ABC, AD \perp BC$, such that $AD^2 = BD \times CD$. Prove that ΔABC is right angled at A.

Ans : [Board Term-1 2015]

As per given condition we have drawn the figure below.



We have $\quad AD^2 = BD \times CD$

$$\frac{AD}{CD} = \frac{BD}{AD}$$

Since $\angle D = 90^\circ$, by SAS we have

$$\Delta ADC \sim \Delta BDA$$

and $\quad \angle BAD = \angle ACD;$

Since corresponding angles of similar triangles are equal

$$\angle DAC = \angle DBA$$

$$\angle BAD + \angle ACD + \angle DAC + \angle DBA = 180^\circ$$

$$2\angle BAD + 2\angle DAC = 180^\circ$$

$$\angle BAD + \angle DAC = 90^\circ$$

$$\angle A = 90^\circ$$

Thus ΔABC is right angled at A.

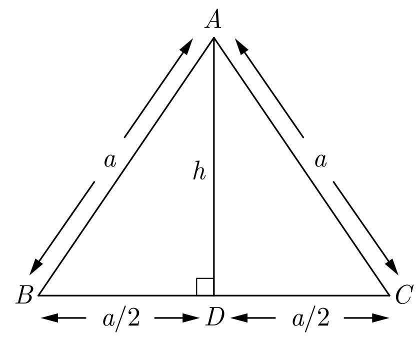
46. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes. [Board 2020 SQP Standard]

or

Find the altitude of an equilateral triangle when each of its side is a cm.

Ans : [Board Term-1 2016]

Let ΔABC be an equilateral triangle of side a and AD is altitude which is also a perpendicular bisector of side BC. This is shown in figure given below.



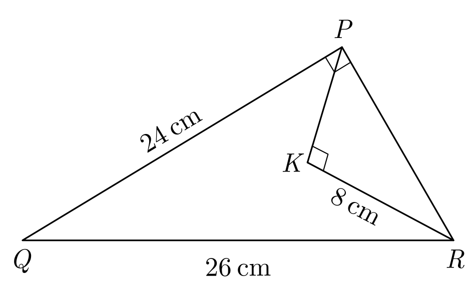
In ΔABD, $\quad a^2 = \left(\frac{a}{2}\right)^2 + h^2$

$$h^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$$

Thus $\quad h = \frac{\sqrt{3}a}{2}$

Thus $\quad 4h^2 = 3a^2 \quad$ Hence Proved

47. In the given triangle $PQR, \angle QPR = 90^\circ, PQ = 24$ cm and $QR = 26$ cm and in $\Delta PKR, \angle PKR = 90^\circ$ and $KR = 8$ cm, find PK.



Ans : [Board Term-1 2012]

In the given triangle we have

$$\angle QPR = 90^0$$

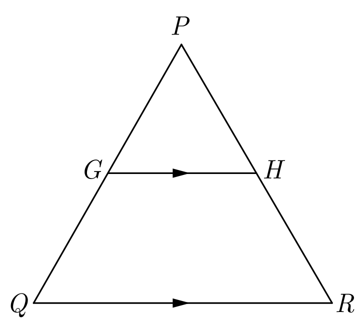
Thus $\quad QR^2 = QP^2 + PR^2$

$$PR = \sqrt{26^2 - 24^2}$$
$$= \sqrt{100} = 10 \text{ cm}$$

Now $\angle PKR = 90^0$

Thus $PK = \sqrt{10^2 - 8^2} = \sqrt{100-64}$
$$= \sqrt{36} = 6 \text{ cm}$$

48. In the given figure, G is the mid-point of the side PQ of ΔPQR and $GH || QR$. Prove that H is the mid-point of the side PR or the triangle PQR.



Ans : [Board Term-1 2012]

Since G is the mid-point of PQ we have

$$PG = GQ$$
$$\frac{PG}{GQ} = 1$$

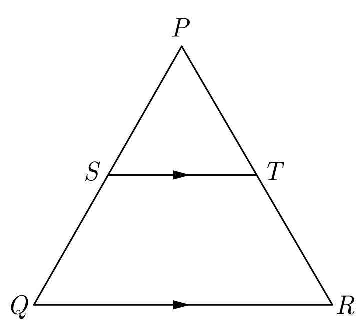
We also have $GH || QR$, thus by BPT we get

$$\frac{PG}{GQ} = \frac{PH}{HR}$$
$$1 = \frac{PH}{HR}$$

$PH = HR.$ Hence proved.

Hence, H is the mid-point of PR.

49. In the given figure, in a triangle PQR, $ST || QR$ and $\frac{PS}{SQ} = \frac{3}{5}$ and $PR = 28$ cm, find PT.



Ans : [Board Term-1 2011]

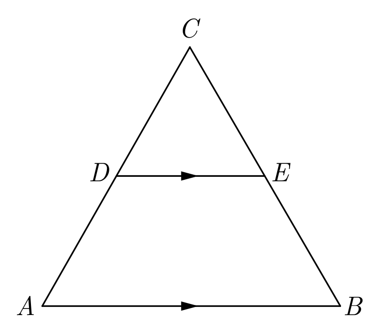
We have $\frac{PS}{SQ} = \frac{3}{5}$

$$\frac{PS}{PS+SQ} = \frac{3}{3+5}$$
$$\frac{PS}{PQ} = \frac{3}{8}$$

We also have, $ST || QR$, thus by BPT we get

$$\frac{PS}{PQ} = \frac{PT}{PR}$$
$$PT = \frac{PS}{PQ} \times PR$$
$$= \frac{3 \times 28}{8} = 10.5 \text{ cm}$$

50. In the given figure, $\angle A = \angle B$ and $AD = BE$. Show that $DE || AB$.



Ans : [Board Term-1, 2012, set-63]

In ΔCAB, we have

$$\angle A = \angle B \quad (1)$$

By isosceles triangle property we have

$$AC = CB$$

But, we have been given

$$AD = BE \quad (2)$$

Dividing equation (2) by (1) we get,

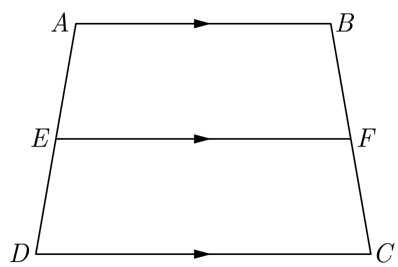
$$\frac{CD}{AD} = \frac{CE}{BE}$$

By converse of BPT,

$DE || AB.$ Hence Proved

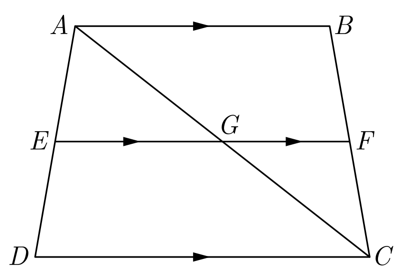
51. In the given figure, if $ABCD$ is a trapezium in which

$AB || CD || EF$, then prove that $\frac{AE}{ED} = \frac{BF}{FC}$



Ans : [Board Term-1 2012]

We draw, AC intersecting EF at G as shown below.



In $\Delta\, CAB$, $GF || AB$, thus by BPT we have

$$\frac{AG}{CG} = \frac{BF}{FC} \qquad ...(1)$$

In $\Delta\, ADC$, $EG || DC$, thus by BPT we have

$$\frac{AE}{ED} = \frac{AG}{CG} \qquad ...(2)$$

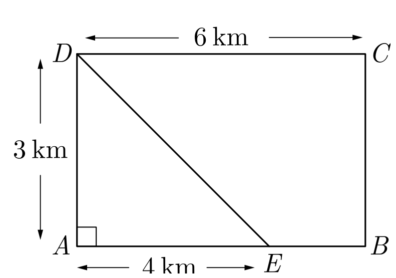
From equations (1) and (2),

$$\frac{AE}{ED} = \frac{BF}{FC}.$$ Hence Proved.

52. In a rectangle $ABCD, E$ is a point on AB such that $AE = \frac{2}{3}AB$. If $AB = 6$ km and $AD = 3$ km, then find DE.

Ans : [Board Term-1 2016]

As per given condition we have drawn the figure below.



We have $$AE = \frac{2}{3}AB = \frac{2}{3} \times 6 = 4 \text{ km}$$

In right triangle ADE,

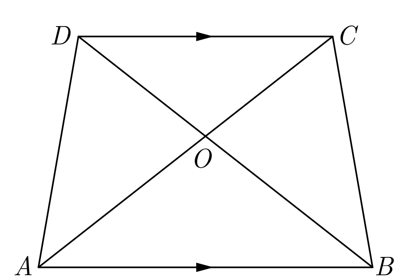
$$DE^2 = (3)^2 + (4)^2 = 25$$

Thus $$DE = 5 \text{ km}$$

53. $ABCD$ is a trapezium in which $AB || CD$ and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Ans : [Board Term-1 2012]

As per given condition we have drawn the figure below.



In $\Delta\, AOB$ and $\Delta\, COD$, $AB \parallel CD$,

Thus due to alternate angles

$$\angle OAB = \angle DCO$$

and $$\angle OBA = \angle ODC$$

By AA similarity we have

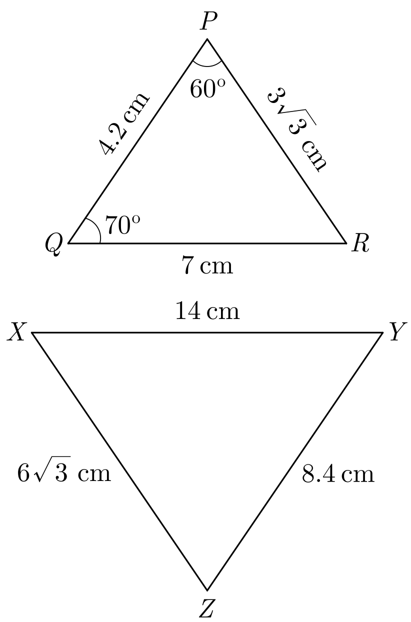
$$\Delta AOB \sim \Delta COD$$

For corresponding sides of similar triangles we have

$$\frac{AO}{CO} = \frac{BO}{DO}$$

$$\frac{AO}{BO} = \frac{CO}{DO}. \qquad \text{Hence Proved}$$

54. In the given figures, find the measure of $\angle X$.



Ans : [Board Term-1 2012]

From given figures,

$$\frac{PQ}{ZY} = \frac{4.2}{8.4} = \frac{1}{2},$$

$$\frac{PR}{ZX} = \frac{3\sqrt{3}}{6\sqrt{3}} = \frac{1}{2}$$

and $$\frac{QR}{YX} = \frac{7}{14} = \frac{1}{2}$$

Thus $$\frac{QP}{ZY} = \frac{PR}{ZX} = \frac{QR}{YX}$$

By SSS criterion we have

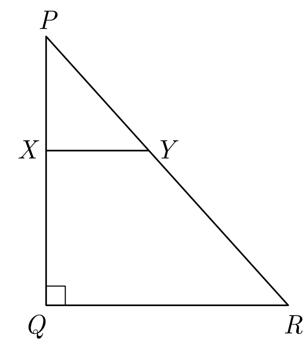
$$\Delta PQR \sim \Delta ZYX$$

Thus $$\angle X = \angle R$$

$$= 180^\circ - (60^\circ + 70^\circ) = 50^\circ$$

Thus $\angle X = 50^\circ$

55. In the given figure, PQR is a triangle right angled at Q and $XY || QR$. If $PQ = 6$ cm, $PY = 4$ cm and $PX : XQ = 1:2$. Calculate the length of PR and QR.



Ans : [Board Term-1 2012]

Since $XY \,||\, OR$, by BPT we have

$$\frac{PX}{XQ} = \frac{PY}{YR}$$

$$\frac{1}{2} = \frac{PY}{PR - PY}$$

$$= \frac{4}{PR - 4}$$

$$PR - 4 = 8 \Rightarrow PR = 12 \text{ cm}$$

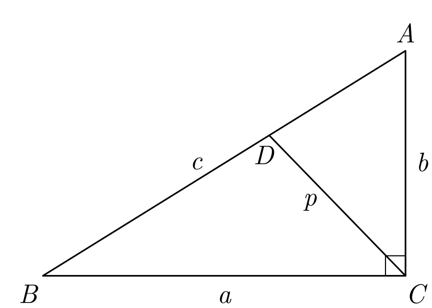
In right ΔPQR we have

$$QR^2 = PR^2 - PQ^2$$

$$= 12^2 - 6^2 = 144 - 36 = 108$$

Thus $QR = 6\sqrt{3}$ cm

56. ABC is a right triangle right angled at C. Let $BC = a$, $CA = b, AB = c$ $PQR, ST || QR$ and p be the length of perpendicular from C to AB. Prove that $cp = ab$.



Ans : [Board Term-1 2012]

In the given figure $CD \perp AB$, and $CD = p$

Area, $$\Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times AB \times CD = \frac{1}{2} cp$$

Also,Area of $\Delta ABC = \frac{1}{2} \times BC \times AC = \frac{1}{2} ab$

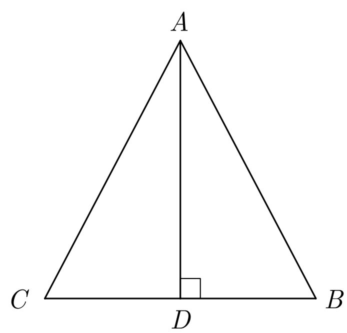
Thus $\frac{1}{2} cp = \frac{1}{2} ab$

$$cp = ab$$ Proved

57. In an equilateral triangle ABC, AD is drawn perpendicular to BC meeting BC in D. Prove that $AD^2 = 3BD^2$.

Ans : [Board Term-1 2012]

In ΔABD, from Pythagoras theorem,



$$AB^2 = AD^2 + BD^2$$

Since $AB = BC = CA$, we get

$$BC^2 = AD^2 + BD^2,$$

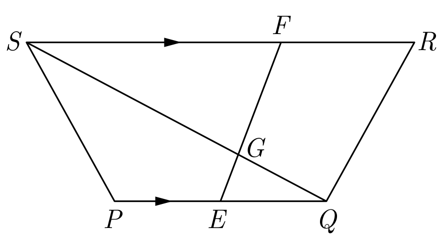
Since $\perp$ is the median in an equilateral Δ, $BC = 2BD$

$$(2BD)^2 = AD^2 + BD^2$$

$$4BD^2 - BD^2 = AD^2$$

$$3BD^2 = AD^2$$

58. In the figure, $PQRS$ is a trapezium in which $PQ || RS$. On PQ and RS, there are points E and F respectively such that EF intersects SQ at G. Prove that $EQ \times GS = GQ \times FS$.



Ans : [Board Term-1 2016]

In ΔGEQ and ΔGFS,

Due to vertical opposite angle,

$$\angle EGQ = \angle FGS$$

Due to alternate angle,

$$\angle EQG = \angle FSG$$

Thus by AA similarity we have

$$\Delta GEQ \sim GFS$$

$$\frac{EQ}{FS} = \frac{GQ}{GS}$$

$$EQ \times GS = GQ \times FS$$

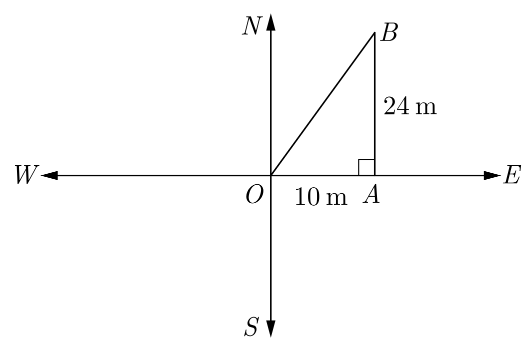
59. A man steadily goes 10 m due east and then 24 m due north.

(1) Find the distance from the starting point.

(2) Which mathematical concept is used in this problem?

Ans :

(1) Let the initial position of the man be at O and his final position be B. The man goes to 10 m due east and then 24 m due north. Therefore, ΔAOB is a right triangle right angled at A such that $OA = 10$ m and $AB = 24$ m. We have shown this condition in figure below.



By Pythagoras theorem,

$$OB^2 = OA^2 + AB^2$$

$$= (10)^2 + (24)^2$$

$$= 100 + 576 = 676$$

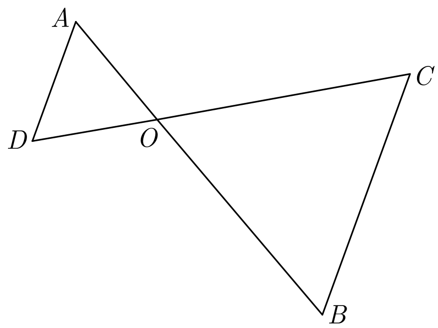
or, $OB = \sqrt{676} = 26$ m

Hence, the man is at a distance of 26 m from the starting point.

(2) Pythagoras Theorem

60. In the given figure, $OA \times OB = OC \times OD$, show that

$\angle A = \angle C$ and $\angle B = \angle D$.



Ans : [Board Term-1 2012]

We have $OA \times OB = OC \times OD$

$$\frac{OA}{OD} = \frac{OC}{OB}$$

Due to the vertically opposite angles,

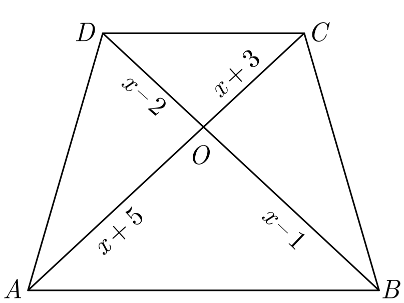
$$\angle AOD = \angle COB$$

Thus by SAS similarity we have

$$\Delta AOD \sim \Delta COB$$

Thus $\angle A = \angle C$ and $\angle B = \angle D$. because of corresponding angles of similar triangles.

61. In the given figure, if $AB || DC$, find the value of x.



Ans : [Board Term-1 2012]

We know that diagonals of a trapezium divide each other proportionally. Therefore

$$\frac{OA}{OC} = \frac{BO}{OD}$$

$$\frac{x+5}{x+3} = \frac{x-1}{x-2}$$

$$(x+5)(x-2) = (x-1)(x+3)$$

$$x^2 - 2x + 5x - 10 = x^2 + 3x - x - 3$$

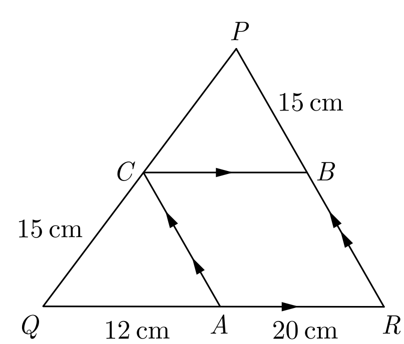
$$x^2 + 3x - 10 = x^2 + 2x - 3$$

$$3x - 10 = 2x - 3$$

$$3x - 2x = 10 - 3 \Rightarrow x = 7$$

Thus $x = 7$.

62. In the given figure, $CB || QR$ and $CA || PR$. If $AQ = 12$ cm, $AR = 20$ cm, $PB = CQ = 15$ cm, calculate PC and BR.



Ans : [Board Term-1 2012]

In ΔPQR, $CA || PR$

By BPT similarity we have

$$\frac{PC}{CQ} = \frac{RA}{AQ}$$

$$\frac{PC}{15} = \frac{20}{12}$$

$$PC = \frac{15 \times 20}{12} = 25 \text{ cm}$$

In ΔPQR, $CB \parallel QR$

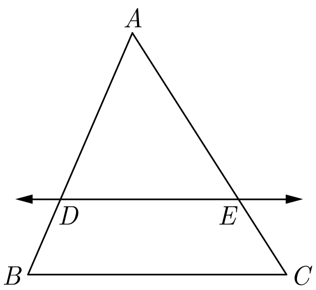
Thus $\frac{PC}{CQ} = \frac{PR}{BR}$

$$\frac{25}{15} = \frac{15}{BR}$$

$$BR = \frac{15 \times 15}{25} = 9 \text{ cm}$$

THREE MARKS QUESTIONS

63. In Figure, in ΔABC, $DE \parallel BC$ such that $AD = 2.4$ cm, $AB = 3.2$ cm and $AC = 8$ cm, then what is the length of AE?



Ans : [Board 2020 Delhi Basic]

We have $DE \parallel BC$

By BPT, $\frac{AD}{DB} = \frac{AE}{EC}$

$$\frac{2.4}{AB - AD} = \frac{AE}{AC - AE}$$

$$\frac{2.4}{3.2 - 2.4} = \frac{AE}{8 - AE}$$

$$\frac{2.4}{0.8} = \frac{AE}{8 - AE}$$

$$3 = \frac{AE}{8 - AE}$$

$$\frac{3}{1+3} = \frac{AE}{8 - AE + AE}$$

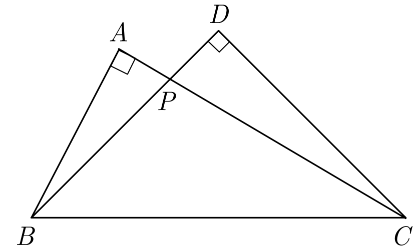
$$\frac{3}{4} = \frac{AE}{8} \Rightarrow AE = 6 \text{ cm}$$

64. Two right triangles ABC and DBC are drawn on the same hypotenuse BC and on the same side of BC. If AC and BD intersect at P, prove that $AP \times PC = BP \times DP$.

Ans : [Board 2019 OD]

Let ΔABC, and ΔDBC be right angled at A and D respectively.

As per given information in question we have drawn the figure given below.



In ΔBAP and ΔCDP we have

$$\angle BAP = \angle CDP = 90°$$

and due to vertical opposite angle

$$\angle BPA = \angle CPD$$

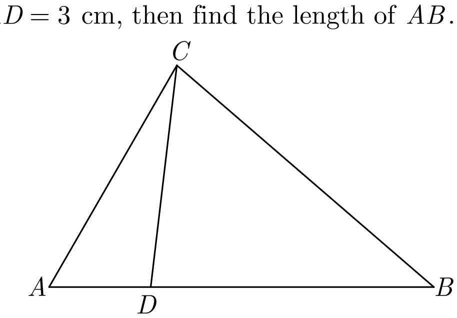
By AA similarity we have

$$\Delta BAP \sim \Delta CDP$$

Therefore $\frac{BP}{PC} = \frac{AP}{PD}$

$$AP \times PC = BP \times PD$$ Hence Proved

65. In the given figure, if $\angle ACB = \angle CDA$, $AC = 6$ cm and $AD = 3$ cm, then find the length of AB.



Ans : [Board 2020 SQP Standard]

In ΔABC and ΔACD we have

$\angle ACB = \angle CDA$ [given]

$\angle CAB = \angle CAD$ [common]

By AA similarity criterion we get

$$\Delta ABC \sim \Delta ACD$$

Thus $\frac{AB}{AC} = \frac{BC}{CD} = \frac{AC}{AD}$

Now $\frac{AB}{AC} = \frac{AC}{AD}$

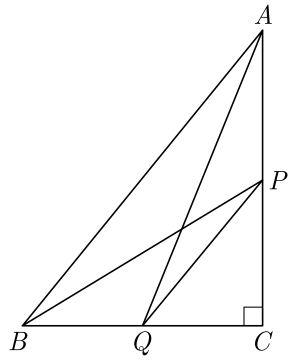
$$AC^2 = AB \times AD$$

$$6^2 = AB \times 3$$

$$AB = \frac{36}{3} = 12 \text{ cm}$$

66. If P and Q are the points on side CA and CB

respectively of ΔABC, right angled at C, prove that $(AQ^2 + BP^2) = (AB^2 + PQ^2)$



Ans : [Board 2019 Delhi]

In right angled triangles ACQ and PCB

$$AQ^2 = AC^2 + CQ^2 \qquad ...(1)$$

and $$BP^2 = PC^2 + CB^2 \qquad ...(2)$$

Adding eq (1) and eq (2), we get

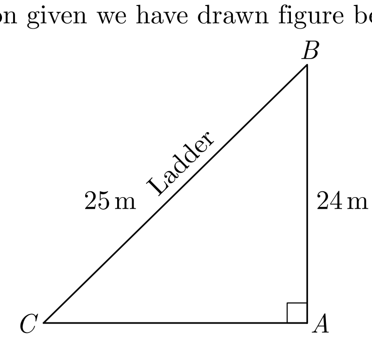
$$AQ^2 + BP^2 = (AC^2 + CQ^2) + (PC^2 + CB^2)$$
$$= (AC^2 + CB^2) + (PC^2 + CQ^2)$$

Thus $$AQ^2 + BP^2 = AB^2 + PQ^2$$ Hence Proved

67. A ladder 25 m long just reaches the top of a building 24 m high from the ground. What is the distance of the foot of ladder from the base of the building?

Ans : [Board 2020 OD Basic]

Let AB be the building and CB be the ladder. As per information given we have drawn figure below.



Here $$AB = 24 \text{ m}$$
$$CB = 25 \text{ m}$$

and $$\angle CAB = 90^\circ$$

By Pythagoras Theorem,

$$CB^2 = AB^2 + CA^2$$

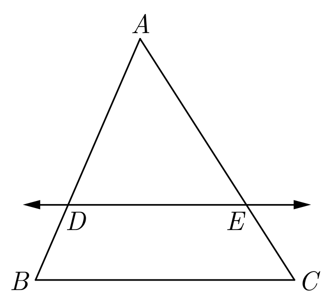
or, $$CA^2 = CB^2 - AB^2$$
$$= 23^2 - 24^2$$
$$= 625 - 576 = 49$$

Thus $$CA = 7 \text{ m}$$

Hence, the distance of the foot of ladder from the building is 7 m.

THREE MARKS QUESTIONS

68. In Figure, in ΔABC, $DE \parallel BC$ such that $AD = 2.4$ cm, $AB = 3.2$ cm and $AC = 8$ cm, then what is the length of AE?



Ans : [Board 2020 Delhi Basic]

We have $$DE \parallel BC$$

By BPT, $$\frac{AD}{DB} = \frac{AE}{EC}$$

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$$\frac{2.4}{AB - AD} = \frac{AE}{AC - AE}$$

$$\frac{2.4}{3.2 - 2.4} = \frac{AE}{8 - AE}$$

$$\frac{2.4}{0.8} = \frac{AE}{8 - AE}$$

$$3 = \frac{AE}{8 - AE}$$

$$\frac{3}{1+3} = \frac{AE}{8 - AE + AE}$$

$$\frac{3}{4} = \frac{AE}{8} \Rightarrow AE = 6 \text{ cm}$$

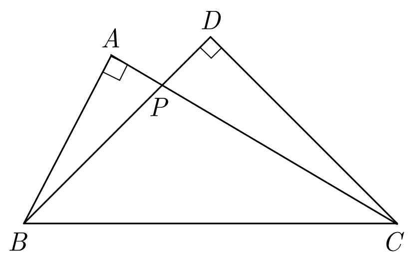
69. Two right triangles ABC and DBC are drawn on the same hypotenuse BC and on the same side of BC. If AC and BD intersect at P, prove that

$AP \times PC = BP \times DP$.

Ans : [Board 2019 OD]

Let ΔABC, and ΔDBC be right angled at A and D respectively.

As per given information in question we have drawn the figure given below.



f243

In $\triangle BAP$ and $\triangle CDP$ we have

$$\angle BAP = \angle CDP = 90^\circ$$

and due to vertical opposite angle

$$\angle BPA = \angle CPD$$

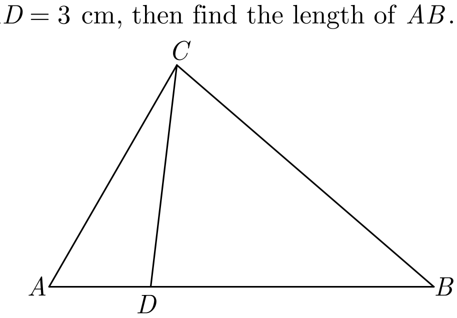
By AA similarity we have

$$\triangle BAP \backsim \triangle CDP$$

Therefore $$\frac{BP}{PC} = \frac{AP}{PD}$$

$$AP \times PC = BP \times PD \qquad \text{Hence Proved}$$

70. In the given figure, if $\angle ACB = \angle CDA$, $AC = 6$ cm and $AD = 3$ cm, then find the length of AB.



Ans : [Board 2020 SQP Standard]

In ΔABC and ΔACD we have

$$\angle ACB = \angle CDA \qquad \text{[given]}$$

$$\angle CAB = \angle CAD \qquad \text{[common]}$$

By AA similarity criterion we get

$$\Delta ABC \sim \Delta ACD$$

Thus $$\frac{AB}{AC} = \frac{BC}{CD} = \frac{AC}{AD}$$

f245

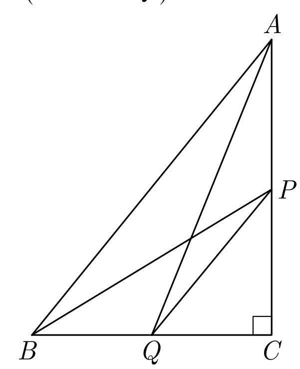
Now $$\frac{AB}{AC} = \frac{AC}{AD}$$

$$AC^2 = AB \times AD$$

$$6^2 = AB \times 3$$

$$AB = \frac{36}{3} = 12 \text{ cm}$$

71. If P and Q are the points on side CA and CB respectively of ΔABC, right angled at C, prove that $(AQ^2 + BP^2) = (AB^2 + PQ^2)$



f246

Ans : [Board 2019 Delhi]

In right angled triangles ACQ and PCB

$$AQ^2 = AC^2 + CQ^2 \qquad ...(1)$$

and $$BP^2 = PC^2 + CB^2 \qquad ...(2)$$

Adding eq (1) and eq (2), we get

$$AQ^2 + BP^2 = (AC^2 + CQ^2) + (PC^2 + CB^2)$$

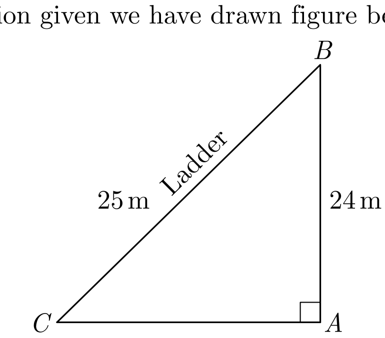
$$= (AC^2 + CB^2) + (PC^2 + CQ^2)$$

Thus $$AQ^2 + BP^2 = AB^2 + PQ^2 \qquad \text{Hence Proved}$$

72. A ladder 25 m long just reaches the top of a building 24 m high from the ground. What is the distance of the foot of ladder from the base of the building?

Ans : [Board 2020 OD Basic]

Let AB be the building and CB be the ladder. As per information given we have drawn figure below.



f247

Here $$AB = 24 \text{ m}$$

$$CB = 25 \text{ m}$$

and $$\angle CAB = 90^\circ$$

By Pythagoras Theorem,

$$CB^2 = AB^2 + CA^2$$

or,
$$CA^2 = CB^2 - AB^2$$
$$= 25^2 - 24^2$$
$$= 625 - 576 = 49$$

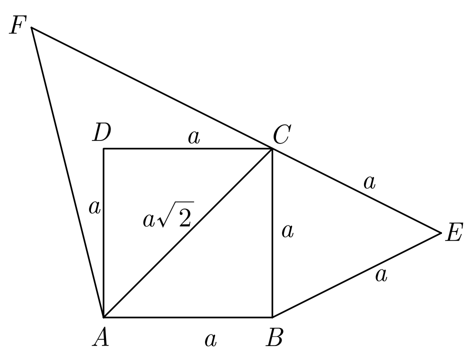
Thus $CA = 7$ m

Hence, the distance of the foot of ladder from the building is 7 m.

73. Prove that area of the equilateral triangle described on the side of a square is half of this area of the equilateral triangle described on its diagonal.

Ans : [Board 2018, 2015]

As per given condition we have drawn the figure below. Let a be the side of square.



By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$
$$= a^2 + a^2 = 2a^2$$
$$AC = \sqrt{2}\,a$$

Area of equilateral triangle ΔBCE,

$$\text{area}(\Delta BCE) = \frac{\sqrt{3}}{4}a^2$$

Area of equilateral triangle ΔACF,

$$\text{area}(\Delta ACF) = \frac{\sqrt{3}}{4}(\sqrt{2}\,a)^2 = \frac{\sqrt{3}}{2}a^2$$

Now, $$\frac{\text{area}(\Delta ACF)}{\text{area}(\Delta BCE)} = 2$$

$$\text{area}(\Delta ACF) = 2\text{area}(\Delta BEC)$$

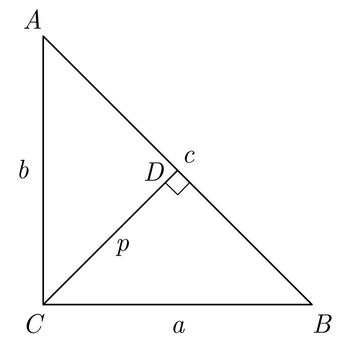
$$\text{area}(\Delta BEC) = \frac{1}{2}\text{area}(\Delta ACF)$$ Hence Proved.

74.

75. ΔABC is right angled at C. If p is the length of the perpendicular from C to AB and a, b, c are the lengths of the sides opposite $\angle A, \angle B$ and $\angle C$ respectively, then prove that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

Ans : [Board Term-1 2016]

As per given condition we have drawn the figure below.



In ΔACB and ΔCDB, $\angle B$ is common and

$$\angle ABC = \angle CDB = 90^0$$

Because of AA similarity we have

$$\Delta ABC \sim \Delta CDB$$

Now $$\frac{b}{p} = \frac{c}{a}$$

$$\frac{1}{p} = \frac{c}{ab}$$

$$\frac{1}{p^2} = \frac{c^2}{a^2b^2}$$

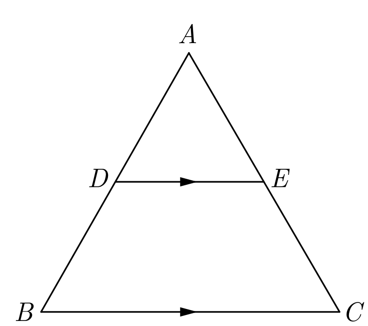
$$\frac{1}{p^2} = \frac{a^2+b^2}{a^2b^2} \qquad (c^2 = a^2 + b^2)$$

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$ Hence Proved

76. In $\Delta ABC, DE || BC$. If $AD = x+2, DB = 3x+16$, $AE = x$ and $EC = 3x+5$, them find x.

Ans : [Board Term-1 2015]

As per given condition we have drawn the figure below.



In the give figure

$$DE \parallel BC$$

By BPT we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x+2}{3x+16} = \frac{x}{x3+5}$$

$$(x+2)(3x+5) = x(3x+16)$$

$$3x^2+5x+6x+10 = 3x^2+16x$$

$$11x+10 = 16x$$

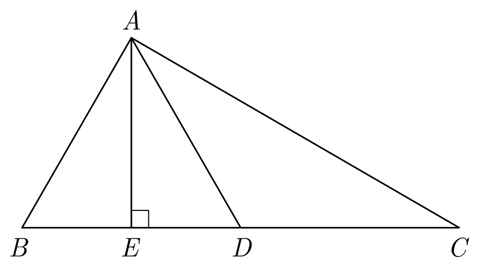
$$11x+10 = 10$$

$$5x = 10 \Rightarrow x = 2$$

77. If in ΔABC, AD is median and $AE \perp BC$, then prove that $AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$.

Ans : [Board Term-1 2015]

As per given condition we have drawn the figure below.



In ΔABE, using Pythagoras theorem we have

$$AB^2 = AE^2 + BE^2$$

$$= AD^2 - DE^2 + (BD - DE)^2$$

$$= AD^2 - DE^2 + BD^2 + DE^2 - 2BD \times DE$$

$$= AD^2 + BD^2 - 2BD \times DE \qquad ...(1)$$

In ΔAEC, we have

$$AC^2 = AE^2 + EC^2$$

$$= (AD^2 - ED^2) + (ED + DC)^2$$

$$= AD^2 - ED^2 + ED^2 + DC^2 + 2ED \times DC$$

$$= AD^2 + CD^2 + 2ED \times CD$$

$$= AD^2 + DC^2 + 2DC \times DE \qquad ...(2)$$

Adding equation (1) and (2) we have

$$AB^2 + AC^2 = 2(AD^2 + BD^2) \qquad (BD = DC)$$

$$= 2AD^2 + 2\left(\frac{1}{2}BC\right)^2 \qquad \left(BD = \tfrac{1}{2}BC\right)$$

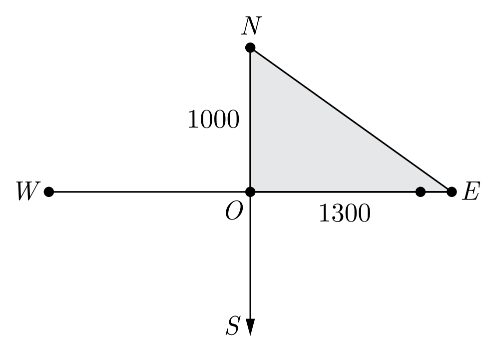
$$= 2AD^2 + \frac{1}{2}BC^2$$

Hence Proves

78. From an airport, two aeroplanes start at the same time. If speed of first aeroplane due North is 500 km/h and that of other due East is 650 km/h then find the distance between the two aeroplanes after 2 hours.

Ans : [Board Term-1 2015]

As per given condition we have drawn the figure below.



Distance covered by first aeroplane due North after two hours,

$$y = 500 \times 2 = 1{,}000 \text{ km.}$$

Distance covered by second aeroplane due East after two hours,

$$x = 650 \times 2 = 1{,}300 \text{ km.}$$

Distance between two aeroplane after 2 hours

$$NE = \sqrt{ON^2 + OE^2}$$

$$= \sqrt{(1000)^2 + (1300)^2}$$

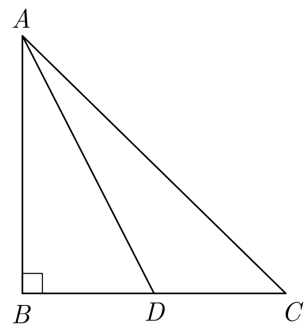
$$= \sqrt{1000000 + 1690000}$$

$$= \sqrt{2690000}$$

$$= 1640.12 \text{ km}$$

79. In the given figure, ABC is a right angled triangle, $\angle B = 90^\circ$. D is the mid-point of BC. Show that

$AC^2 = AD^2 + 3CD^2$.



Ans : [Board Term-1 2016]

We have $BD = CD = \frac{BC}{2}$

$$BC = 2BD$$

Using Pythagoras theorem in the right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$

$$= AB^2 + (2BD)$$

$$= AB^2 + 4BD^2$$

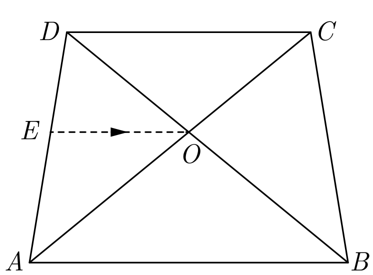
$$= (AB^2 + BD^2) + 3BD^2$$

$$AC^2 = AD^2 + 3CD^2$$

80. If the diagonals of a quadrilateral divide each other proportionally, prove that it is a trapezium.

Ans : [Board Term-1 2011]

As per given condition we have drawn quadrilateral $ABCD$, as shown below.



We have drawn $EO || AB$ on DA.

In quadrilateral $ABCD$, we have

$$\frac{AO}{BO} = \frac{CO}{DO}$$

$$\frac{AO}{CO} = \frac{BO}{DO} \qquad ...(1)$$

In ΔABD, $EO || AB$

By BPT we have

$$\frac{AE}{ED} = \frac{BO}{DO} \qquad ...(2)$$

From equation (1) and (2), we get

$$\frac{AE}{ED} = \frac{AO}{CO}$$

In ΔADC, $\frac{AE}{ED} = \frac{AO}{CO}$

$EO || DC$ (Converse of BPT)

$EO || AB$ (Construction)

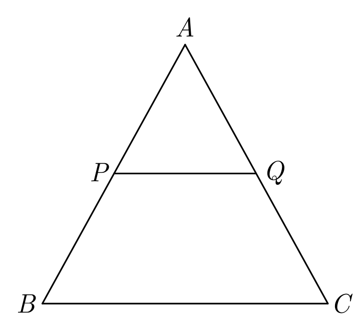
$AB || DC$

Thus in quadrilateral $ABCD$ we have

$$AB \; AB \parallel CD$$

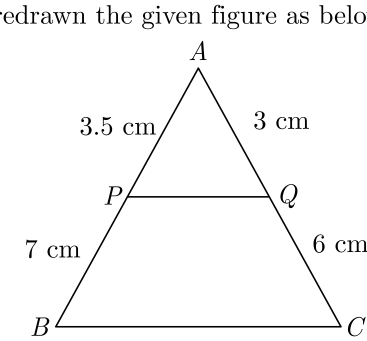
Thus $ABCD$ is a trapezium. Hence Proved

81. In the given figure, P and Q are the points on the sides AB and AC respectively of ΔABC, such that $AP = 3.5$cm, $PB = 7$ cm, $AQ = 3$ cm and $QC = 6$ cm. If $PQ = 4.5$ cm, find BC.



Ans : [Board Term-1 2011]

We have redrawn the given figure as below.



We have $\frac{AP}{AB} = \frac{3.5}{10.5} = \frac{1}{3}$

and $\quad \dfrac{AQ}{AC} = \dfrac{3}{9} = \dfrac{1}{3}$

In $\Delta\, ABC$, $\quad \dfrac{AP}{AB} = \dfrac{AQ}{AC}$ and $\angle A$ is common.

Thus due to SAS we have

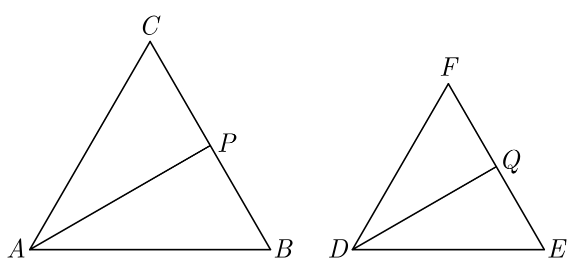
$$\Delta\, APQ \sim \Delta\, ABC$$

$$\frac{AP}{AB} = \frac{PQ}{BC}$$

$$\frac{1}{3} = \frac{4.5}{BC}$$

$$BC = 13.5 \text{ cm.}$$

82. In given figure $\Delta\, ABC \sim \Delta\, DEF$. AP bisects $\angle CAB$ and DQ bisects $\angle FDE$.



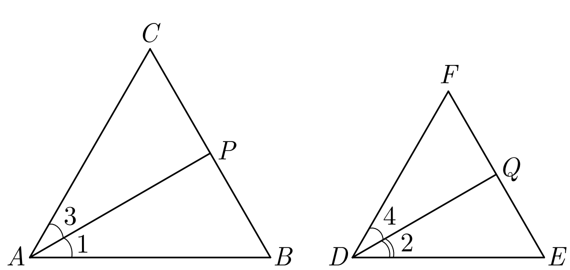
Prove that :

(1) $\dfrac{AP}{DQ} = \dfrac{AB}{DE}$

(2) $\Delta\, CAP \sim \Delta\, FDQ$.

Ans : [Board Term-1 2016]

As per given condition we have redrawn the figure below.



(1) Since $\Delta\, ABC \sim \Delta\, DEF$

$$\angle A = \angle D \quad \text{(Corresponding angles)}$$

$$2\angle 1 = 2\angle 2$$

Also $\quad \angle B = \angle E \quad$ (Corresponding angles)

$$\frac{AP}{DQ} = \frac{AB}{DE} \qquad \text{Hence Proved}$$

(2) Since $\Delta\, ABC \sim \Delta\, DEF$

$$\angle A = \angle D$$

and $\quad \angle C = \angle F$

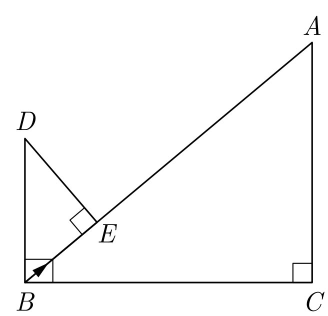
$$2\angle 3 = 2\angle 4$$

$$\angle 3 = \angle 4$$

By AA similarity we have

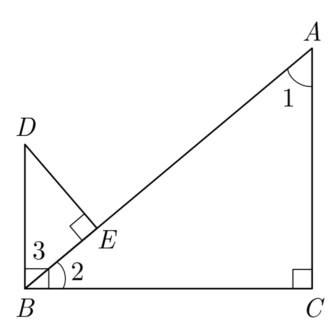
$$\Delta\, CAP \sim \Delta\, FDQ$$

83. In the given figure, $DB \perp BC, DE \perp AB$ and $AC \perp BC$. Prove that $\dfrac{BE}{DE} = \dfrac{AC}{BC}$.



Ans : [Board Term-1 2011]

As per given condition we have redrawn the figure below.



We have $DB \perp BC, DE \perp AB$ and $AC \perp BC$.

In $\Delta\, ABC$, $\angle C = 90^{\circ}$, thus

$$\angle 1 + \angle 2 = 90^{\circ}$$

But we have been given,

$$\angle 2 + \angle 3 = 90^{\circ}$$

Hence $\angle 1 = \angle 3$

In ΔABC and ΔBDE,

$$\angle 1 = \angle 3$$

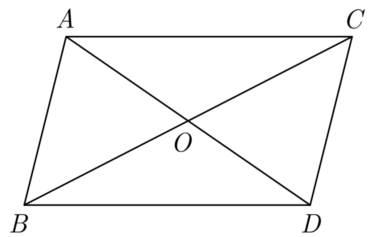
and $\angle ACB = \angle DEB = 90^{\circ}$

Thus by AA similarity we have

$$\Delta ABC \sim \Delta BDE$$

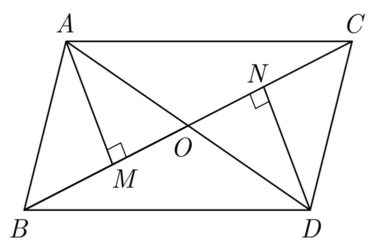
Thus $\dfrac{AC}{BC} = \dfrac{BE}{DE}$. Hence Proved

84. In the given figure, ΔABC and ΔABC and ΔDBC are on the same base BC. AD and BC intersect at O. Prove that $\dfrac{ar(\Delta ABC)}{ar(\Delta DBC)} = \dfrac{AO}{DO}$.



Ans : [Board 2020 OD Std, 2016, 2011]

As per given condition we have redrawn the figure below. Here we have drawn $AM \perp BC$ and $DN \perp BC$.



In ΔAOM and ΔDON,

$\angle AOM = \angle DON$

(Vertically opposite angles)

$\angle AMO = \angle DNO = 90^{\circ}$ (Construction)

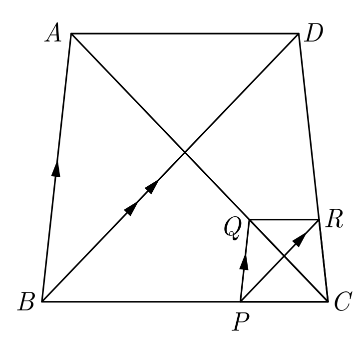
or, $\Delta AOM \sim \Delta DON$ (By AA similarity)

Thus $\dfrac{AO}{DO} = \dfrac{AM}{DN}$ ...(1)

Now, $\dfrac{ar(\Delta ABC)}{ar(\Delta DBC)} = \dfrac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times BC \times DN}$

$= \dfrac{AM}{DN} = \dfrac{AO}{DO}$ From equation (1)

85. In the given figure, two triangles ABC and DBC lie on the same side of BC such that $PQ || BA$ and $PR || BD$. Prove that $QR || AD$.



Ans : [Board Term-1 2011]

In ΔABC, we have $PQ || AB$ and $PR || BD$.

By BPT we have

$$\frac{BP}{PC} = \frac{AQ}{QC} \quad ...(1)$$

Again in ΔBCD, we have

$$PR \,||\, BD$$

By BPT we have

$$\frac{BP}{PC} = \frac{DR}{RC} \qquad \text{(by BPT) ...(2)}$$

$$\frac{AQ}{QC} = \frac{DR}{RC}$$

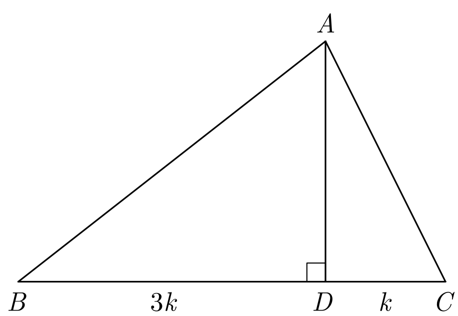
By converse of BPT,

$$PR \parallel AD \qquad \text{Hence proved}$$

86. The perpendicular AD on the base BC of a ΔABC intersects BC at D so that $DB = 3CD$. Prove that $2(AB)^2 = 2(AC)^2 + BC^2$.

Ans : [Board Term-1 2011, 2012, 2016]

As per given condition we have drawn the figure below.



Here $\qquad DB = 3CD$

$$BD = \frac{3}{4}BC$$

$$DC = \frac{1}{4}BC$$

In ΔADB, we have

$$AB^2 = AD^2 + BD^2 \qquad ...(1)$$

In ΔADC, $\qquad AC^2 = AD^2 + CD^2 \qquad ...(2)$

Subtracting equation (2) from (1), we get

$$AB^2 - AC^2 = BD^2 - CD^2$$

Since $DB = 3CD$ we get

$$AB^2 - AC^2 = \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2$$

$$= \frac{9}{16}BC^2 - \frac{1}{16}BC^2 = \frac{BC^2}{2}$$

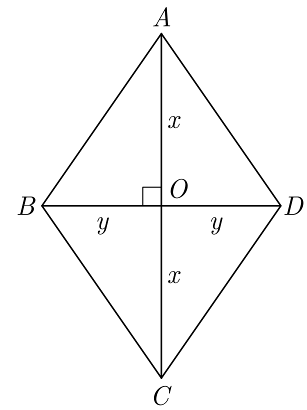
$$2(AB^2 - AC^2) = BC^2$$

$$2(AB)^2 = 2AC^2 + BC^2 \qquad \text{Hence Proved}$$

87. Prove that the sum of squares on the sides of a rhombus is equal to sum of squares of its diagonals.

Ans : [Board Term-1 2011]

Let, $ABCD$ is a rhombus and we know that diagonals of a rhombus bisect each other at 90^o.



Now $\qquad AO = OC \Rightarrow AO^2 \quad OC$

$$BO = OD \Rightarrow BO^2 \quad OD$$

and $\qquad \angle AOB = 90^o$

$$AB^2 = OA^2 + BO^2 = x^2 + y^2$$

Similarly, $\qquad AD^2 = OA^2 + OD^2 = x^2 + y^2$

$$CD^2 = OC^2 + OD^2 = x^2 + y^2$$

$$CB^2 = OC^2 + OB^2 = x^2 + y^2$$

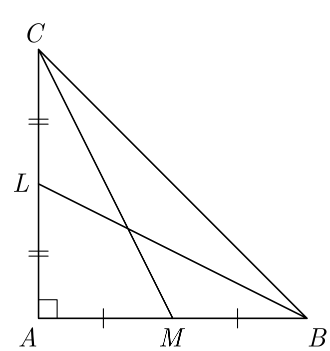
$$AB^2 + BC^2 + CD^2 + DA^2 = 4x^2 + 4y^2$$

$$= (2x)^2 + (2y)^2$$

$$AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$$

Hence Proved

88. In the given figure, BL and CM are medians of ΔABC, right angled at A. Prove that $4(BL^2 + CM^2) = 5BC^2$.



Ans : [Board T

We have a right angled triangle ΔABC at A where BL and CM are medians.

In ΔABL, $BL^2 = AB^2 + AL^2$

$$= AB^2 + \left(\frac{AC}{2}\right)^2 \text{ (}BL\text{ is median)}$$

In ΔACM, $CM^2 = AC^2 + AM^2$

$$= AC^2 + \left(\frac{AB}{2}\right)^2 \text{ (}CM\text{ is median)}$$

Now $BL^2 + CM^2 = AB^2 + AC^2 + \frac{AC^2}{4} + \frac{AB^2}{4}$

$$4(BL^2 + CM^2) = 5AB^2 + 5AC^2$$

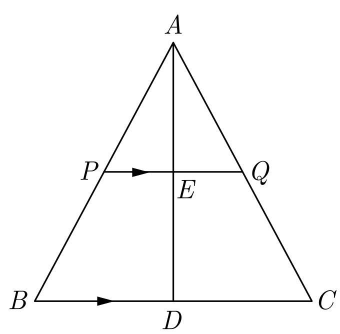
$$= 5(AB^2 + AC^2)$$

$$= 5BC^2$$ Hence Proved

89. In a ΔABC, let P and Q be points on AB and AC respectively such that $PQ || BC$. Prove that the median AD bisects PQ.

Ans : [Board Term-1 2011]

As per given condition we have drawn the figure below.



The median AD intersects PQ at E.

We have, PQ || BE

$\angle ApE = \angle B$ and $\angle AQE$

$= \angle C$

(Corresponding angles)

Thus in ΔAPE and ΔABD we have

$$\angle APE = \angle ABD$$

$$\angle PAE = \angle BAD \quad \text{(common)}$$

Thus $\Delta APE \sim \Delta ABD$

$$\frac{PE}{BD} = \frac{AE}{AD} \quad ...(1)$$

Similarly, $\Delta AQE \sim \Delta ACD$

or, $$\frac{QE}{CD} = \frac{AE}{AD} \quad ...(2)$$

From equation (1) and (2) we have

$$\frac{PE}{BD} = \frac{QE}{CD}$$

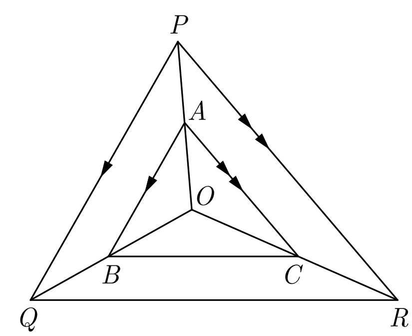
As $CD = BD$, we get

$$\frac{PE}{BD} = \frac{QE}{BD}$$

$$PE = QE$$

Hence, AD bisects PQ.

90. In the given figure A, B and C are points on OP, OQ and OR respectively such that $AB || PQ$ and $AC || PR$. Prove that $BC || QR$.



Ans : [Board Term-1 2012]

In ΔPOQ, $AB \parallel PQ$

By BPT $$\frac{AO}{AP} = \frac{OB}{BQ} \quad ...(1)$$

In ΔOPR, $AC \parallel PR$,

By BPT $$\frac{OA}{AP} = \frac{OC}{CR} \quad (2)$$

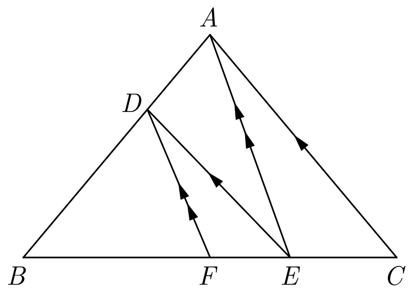
From equations (1) and (2), we have

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

By converse of BPT we have

$BC \parallel QR$ Hence Proved

91. In the given figure, $DE || AC$ and $DF || AE$. Prove that $\frac{BE}{FE} = \frac{BE}{EC}$.



Ans : [Board 2020 Delhi Std, 2012]

In ΔABC, $DE \parallel AC$, (Given)

By BPT $\frac{BD}{DA} = \frac{BE}{EC}$ ...(1)

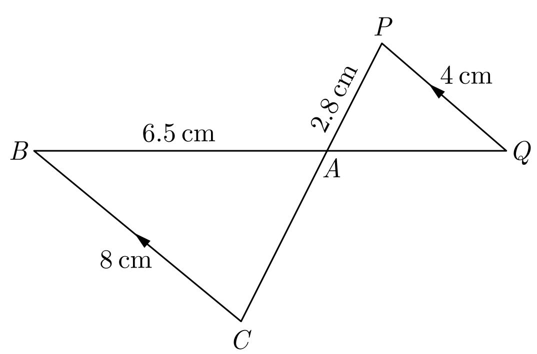
In ΔABE, $DF \parallel AE$, (Given)

By BPT $\frac{BD}{DA} = \frac{BF}{FE}$ ...(2)

From (1) and (2), we have

$$\frac{BF}{FE} = \frac{BE}{EC}.$$

92. In the given figure, $BC || PQ$ and $BC = 8$ cm, $PQ = 4$ cm, $BA = 6.5$ cm $AP = 2.8$ cm Find CA and AQ.



Ans : [Board Term-1 2012]

In ΔABC and ΔAPQ, $AB = 6.5$ cm, $BC = 8$ cm,

$PQ = 4$ cm and $AP = 2.8$ cm.

We have $BC \parallel PQ$

Due to alternate angles

$$\angle CBA = \angle AQP$$

Due to vertically opposite angles,

$$\angle BAC = \angle PAQ$$

Due to AA similarity,

$$\Delta ABC \sim \Delta AQP$$

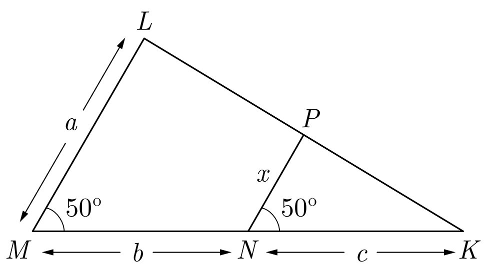
$$\frac{AB}{AQ} = \frac{BC}{QP} = \frac{AC}{AP}$$

$$\frac{6.5}{AQ} = \frac{8}{4} = \frac{AC}{AP}$$

$$AQ = \frac{6.5}{2} = 3.25 \text{ cm}$$

$$AC = 2 \times 2.5 = 5.6 \text{ cm}$$

93. In the given figure, find the value of x in terms of a, b and c.



Ans : [Board Term-1 2012]

In triangles LMK and PNK, $\angle K$ is common and

$$\angle M = \angle N = 50^{\circ}$$

Due to AA similarity,

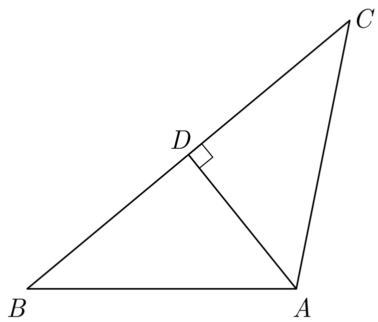
$$\Delta LMK \sim \Delta PNK$$

$$\frac{LM}{PN} = \frac{KM}{KN}$$

$$\frac{a}{x} = \frac{b+c}{c}$$

$$x = \frac{ac}{b+c}$$

94. In the given figure, if $AD \perp BC$, prove that $AB^2 + CD^2 = BD^2 + AC^2$.



Ans : [Board 2020 OD Standard]

In right ΔADC,

$$AC^2 = AD^2 + CD^2 \quad ...(1)$$

In right ΔADB,

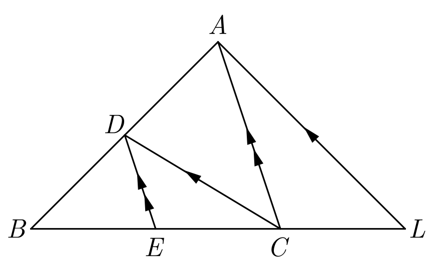
$$AB^2 = AD^2 + BD^2 \quad ...(2)$$

Subtracting equation (1) from (2) we have

$$AB^2 - AC^2 = BD^2 - CD^2$$

$$AB^2 + CD^2 = AC^2 + BD^2.$$

95. In the given figure, $CD || LA$ and $DE || AC$. Find the length of CL, if $BE = 4$ cm and $EC = 2$ cm.



Ans : [Board Term-1 2012]

In ΔABC, $DE || AC$, $BE = 4$ cm and $EC = 2$cm

By BPT $\qquad \dfrac{BD}{DA} = \dfrac{BE}{EC} \qquad ...(1)$

In ΔABL, $\qquad DC || AL$

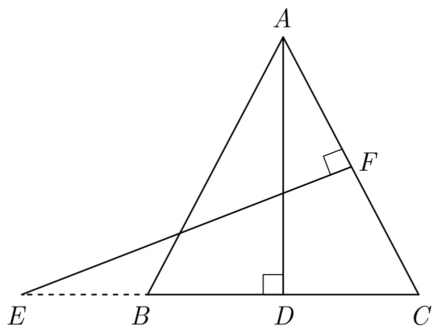
By BPT $\qquad \dfrac{BD}{DA} = \dfrac{BC}{CL} \qquad ...(2)$

From equations (1) and (2),

$$\frac{BE}{EC} = \frac{BC}{CL}$$

$$\frac{4}{2} = \frac{6}{CL} \Rightarrow CL = 3 \text{ cm}$$

96. In the given figure, $AB = AC$. E is a point on CB produced. If AD is perpendicular to BC and EF perpendicular to AC, prove that ΔABD is similar to ΔCEF.



Ans : [Board Term-1 2012]

In ΔABD and ΔCEF, we have

$$AB = AC$$

Thus $\qquad \angle ABC = \angle ACB$

$$\angle ABD = \angle ECF$$

$$\angle ADB = \angle EFC \qquad \text{(each } 90^{\circ}\text{)}$$

Due to AA similarity

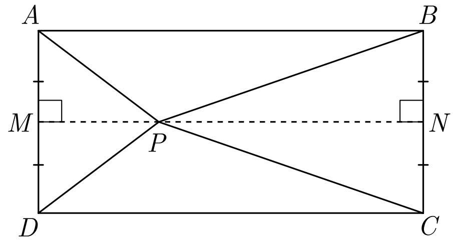
$$\Delta ABD \sim \Delta ECF \qquad \text{Hence proved}$$

FOUR MARKS QUESTIONS

97. In a rectangle $ABCD$, P is any interior point. Then prove that $PA^2 + PC^2 = PB^2 + PD^2$.

Ans : [Board 2020 OD Basic]

As per information given we have drawn figure below.



Here P is any point in the interior of rectangle $ABCD$. We have drawn a line MN through point P and parallel to AB and CD.

We have to prove $PA^2 + PC^2 = PB^2 + PD^2$

Since $AB \parallel MN$, $AM \parallel BN$ and $\angle A = 90°$, thus $ABNM$ is rectangle. $MNCD$ is also a rectangle.

Here, $PM \perp AD$ and $PN \perp BC$,

$$AM = BN \text{ and } MD = NC \quad ...(1)$$

Now, in ΔAMP, $PA^2 = AM^2 + MP^2$ ...(2)

In ΔPMD, $PD^2 = MP^2 + MD^2$ ...(3)

In ΔPNB, $PB^2 = PN^2 + BN^2$ ...(4)

In ΔPNC, $PC^2 = PN^2 + NC^2$ ...(5)

From equation (2) and (5) we obtain,

$$PA^2 + PC^2 = AM^2 + MP^2 + PN^2 + NC^2$$

Using equation (1) we have

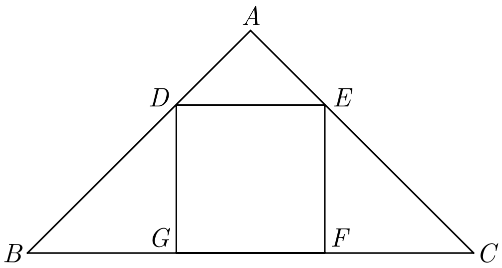
$$PA^2 + PC^2 = BN^2 + MP^2 + PN^2 + MD^2$$

$$= (BN^2 + PN^2) + (MP^2 + MD^2)$$

Using equation (3) and (4) we have

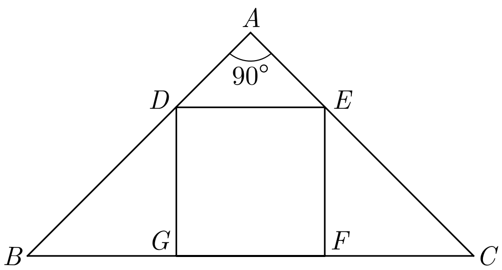
$$PA^2 + PC^2 = PB^2 + PD^2$$

98. In the given figure, $DEFG$ is a square and $\angle BAC = 90°$. Show that $FG^2 = BG \times FC$.



Ans : [Board 2020 SQP Standard]

We have redrawn the given figure as shown below.



In ΔADE and ΔGBD, we have

$$\angle DAE = \angle BGD \quad \text{[each 90°]}$$

Due to corresponding angles we have

$$\angle ADE = \angle GDB$$

Thus by AA similarity criterion,

$$\Delta ADE \sim \Delta GBD$$

Now, in ΔADE and ΔFEC,

$$\angle EAD = \angle CFE \quad \text{[each 90°]}$$

Due to corresponding angles we have

$$\angle AED = \angle FCE$$

Thus by AA similarity criterion,

$$\Delta ADE \sim \Delta FEC$$

Since $\Delta ADE \sim \Delta GBD$ and $\Delta ADE \sim \Delta FEC$ we have

$$\Delta GBD \sim \Delta FEC$$

Thus $$\frac{GB}{FE} = \frac{GD}{FC}$$

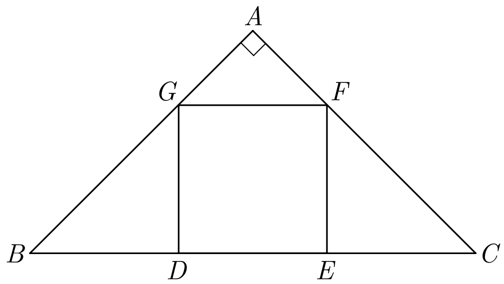
Since $DEFG$ is square, we obtain,

$$\frac{BG}{FG} = \frac{FG}{FC}$$

Therefore $FG^2 = BG \times FC$ Hence Proved

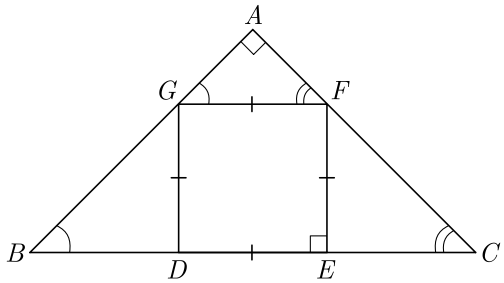
99. In Figure $DEFG$ is a square in a triangle ABC right angled at A. Prove that

(i) $\Delta AGF \sim \Delta DBG$ (ii) $\Delta AGF \sim \Delta EFC$



Ans : [Board 2020 Delhi, OD Basic]

We have redrawn the given figure as shown below.



Here ABC is a triangle in which $\angle BAC = 90°$ and $DEFG$ is a square.

(i) In ΔAGF and ΔDBG

$$\angle GAF = \angle BDG \quad \text{(each 90°)}$$

Due to corresponding angles,

$$\angle AGF = \angle GBD$$

Thus by AA similarity criterion,

$$\Delta AGF \sim \Delta DBG \qquad \text{Hence Proved}$$

(ii) In ΔAGF and ΔEFC,

$$\angle GAF = \angle CEF \qquad \text{(each } 90^\circ\text{)}$$

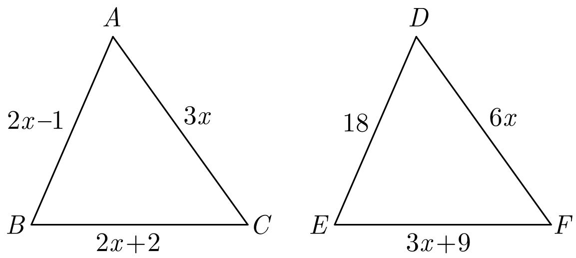
Due to corresponding angles,

$$\angle AFG = \angle FCE$$

Thus by AA similarity criterion,

$$\Delta AGF \sim \Delta EFC \qquad \text{Hence Proved}$$

100. In Figure, if $\Delta ABC \sim \Delta DEF$ and their sides of lengths (in cm) are marked along them, then find the lengths of sides of each triangle.



Ans : [Board 2020 OD Standard]

Since $\Delta ABC \sim \Delta DEF$, we have

$$\frac{AB}{BC} = \frac{DE}{EF}$$

$$\frac{2x-1}{2x+2} = \frac{18}{3x+9}$$

$$(2x-1)(3x+9) = 18(2x+2)$$

$$(2x-1)(x+3) = 6(2x+2)$$

$$2x^2 - x + 6x - 3 = 12x + 12$$

$$2x^2 + 5x - 12x - 15 = 0$$

$$2x^2 - 7x - 15 = 0$$

$$2x^2 - 10x + 3x - 15 = 0$$

$$2x(x-5) + 3(x-5) = 0$$

$$(x-5)(2x+3) = 0 \Rightarrow x = 5 \text{ or } x = \frac{-3}{2}$$

But $x = \frac{-3}{2}$ is not possible, thus $x = 5$.

Now in ΔABC, we get

$$AB = 2x - 1 = 2 \times 5 - 1 = 9$$

$$BC = 2x + 2 = 2 \times 5 + 2 = 12$$

$$AC = 3x = 3 \times 5 = 15$$

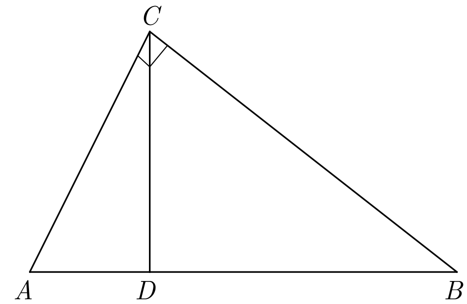
and in ΔDEF, we get

$$DE = 18$$

$$EF = 3x + 9 = 3 \times 5 + 9 = 24$$

$$DE = 6x = 6 \times 5 = 30.$$

101. In Figure , $\angle ACB = 90^\circ$ and $CD \perp AB$, prove that $CD^2 = BD \times AD$.



Ans : [Board 2019 Delhi]

In ΔACB we have

$$\angle ACB = 90^\circ$$

and $$CD \perp AB$$

Thus $$AB^2 = CA^2 + CB^2 \qquad \text{...(1)}$$

In ΔCAD, $\angle ADC = 90^\circ$, thus we have

$$CA^2 = CD^2 + AD^2 \qquad \text{...(2)}$$

and in ΔCDB, $\angle CDB = 90^\circ$, thus we have

$$CB^2 = CD^2 + BD^2 \qquad \text{...(3)}$$

Adding equation (2) and (3), we get

$$CA^2 + CB^2 = 2CD^2 + AD^2 + BD^2$$

Substituting AB^2 from equation (1) we have

$$AB^2 = 2CD^2 + AD^2 + BD^2$$

$$AB^2 - AD^2 = BD^2 + 2CD^2$$

$$(AB + AD)(AB - AD) = BD^2 + 2CD^2$$

$$(AB + AD)BD - BD^2 = 2CD^2$$

$$BD[(AB + AD) - BD] = 2CD^2$$

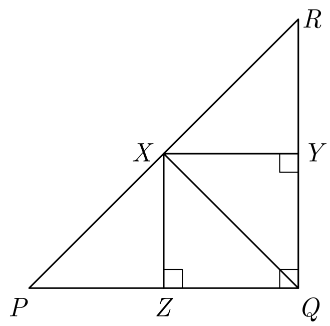
$$BD[AD + (AB - BD)] = 2CD^2$$

$$BD[AD + AD] = 2CD^2$$

$$BD \times 2AD = 2CD^2$$

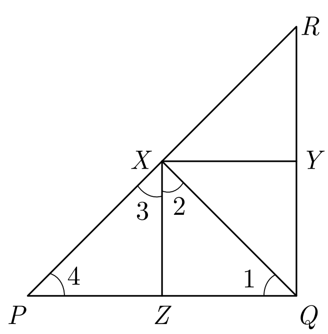
$$CD^2 = BD \times AD \qquad \text{Hence Proved}$$

102. ΔPQR is right angled at Q. $QX \perp PR$, $XY \perp RQ$ and $XZ \perp PQ$ are drawn. Prove that $XZ^2 = PZ \times ZQ$.



Ans : [Board Term-1 2015]

We have redrawn the given figure as below.



It may be easily seen that $RQ \perp PQ$ and $XZ \perp PQ$ or $XZ \parallel YQ$.

Similarly $XY \parallel ZQ$

Since $\angle PQR = 90^\circ$, thus $XYQZ$ is a rectangle.

In ΔXZQ, $\angle 1 + \angle 2 = 90^\circ$ ...(1)

and in ΔPZX, $\angle 3 + \angle 4 = 90^\circ$ ...(2)

$XQ \perp PR$ or, $\angle 2 + \angle 3 = 90^\circ$ ...(3)

From eq. (1) and (3), $\angle 1 = \angle 3$

From eq. (2) and (3), $\angle 2 = \angle 4$

Due to AA similarity,

$$\Delta PZX \sim \Delta XZQ$$

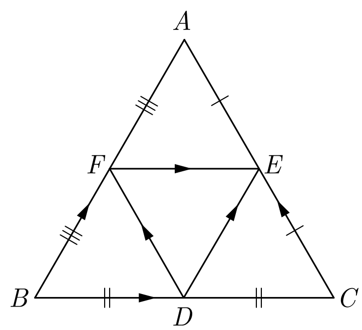
$$\frac{PZ}{XZ} = \frac{XZ}{ZQ}$$

$$XZ^2 = PZ \times ZQ \qquad \text{Hence proved}$$

103. In ΔABC, the mid-points of sides BC, CA and AB are D, E and F respectively. Find ratio of $ar(\Delta DEF)$ to $ar(\Delta ABC.)$

Ans : [Board Term-1 2015]

As per given condition we have given the figure below. Here F, E and D are the mid-points of AB, AC and BC respectively.



Hence, $FE \parallel BC, DE \parallel AB$ and $DF \parallel AC$

By mid-point theorem,

If $DE \parallel BA$ then $DE \parallel BF$

and if $FE \parallel BC$ then $FE \parallel BD$

Therefore $FEDB$ is a parallelogram in which DF is diagonal and a diagonal of parallelogram divides it into two equal Areas.

Hence $ar(\Delta BDF) = ar(\Delta DEF)$ ...(1)

Similarly $ar(\Delta CDE) = ar(\Delta DEF)$ ...(2)

$(\Delta AFE) = ar(\Delta DEF)$ ...(3)

$(\Delta DEF) = ar(\Delta DEF)$ ...(4)

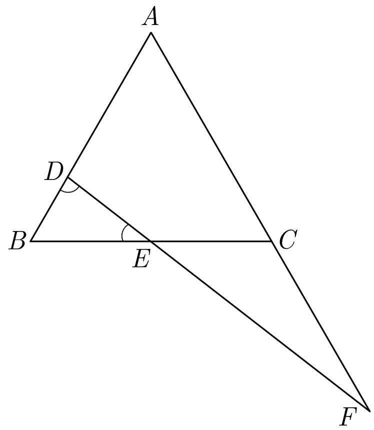
Adding equation (1), (2), (3) and (4), we have

$$ar(\Delta BDF) + ar(\Delta CDE) + ar(\Delta AFE) + ar(\Delta DEF) = 4ar(\Delta DEF)$$

$$ar(\Delta ABC) = 4ar(\Delta DEF)$$

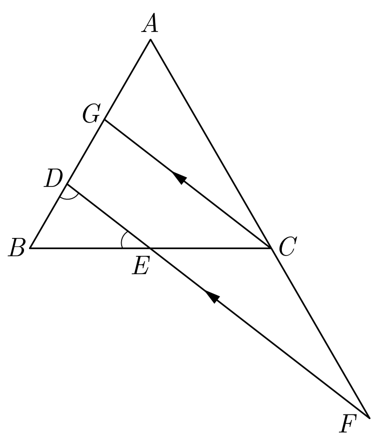
$$\frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$$

104. In the figure, $\angle BED = \angle BDE$ and E is the mid-point of BC. Prove that $\frac{AF}{CF} = \frac{AD}{BE}$.



Ans :

We have redrawn the given figure as below. Here $CG \,||\, FD$.



We have $\qquad \angle BED = \angle BDE$

Since E is mid-point of BC,

$$BE = BD = EC \qquad \text{...(1)}$$

In ΔBCG, $\qquad DE \,||\, FG$

From (1) we have

$$\frac{BD}{DG} = \frac{BE}{EC} = 1$$

$$BD = DG = EC = BE$$

In ΔADF, $\qquad CG \,||\, FD$

By BPT $\qquad \frac{AG}{GD} = \frac{AC}{CF}$

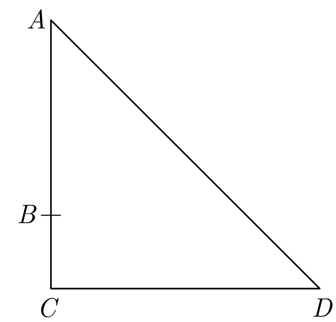
$$\frac{AG + GD}{GD} = \frac{AF + CF}{CF}$$

, $\qquad \frac{AD}{GD} = \frac{AF}{CF}$

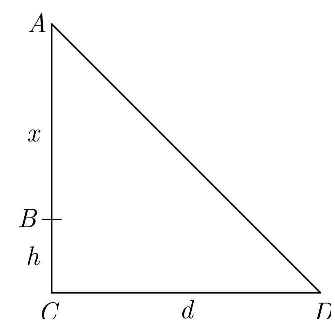
Thus $\qquad \frac{AF}{CF} = \frac{AD}{BE}$

105. In the right triangle, B is a point on AC such that $AB + AD = BC + CD$. If $AB = x, BC = h$ and $CD = d$, then find x (in term of h and d).

Ans : [Board Term-1 2015]



We have redrawn the given figure as below.



We have $\qquad AB + AD = BC + CD$

$$AD = BC + CD - AB$$

$$AD = h + d - x$$

In right ΔACD, we have

$$AD^2 = AC^2 + DC^2$$

$$(h + d - x)^2 = (x + h)^2 + d^2$$

$$(h + d - x)^2 - (x + h)^2 = d^2$$

$$(h + d - x - x - h)(h + d - x + x + h) = d^2$$

$$(d - 2x)(2h + d) = d^2$$

$$2hd + d^2 - 4hx - 2xd = d^2$$

$$2hd = 4hx + 2xd$$

$$= 2(2h + d)x$$

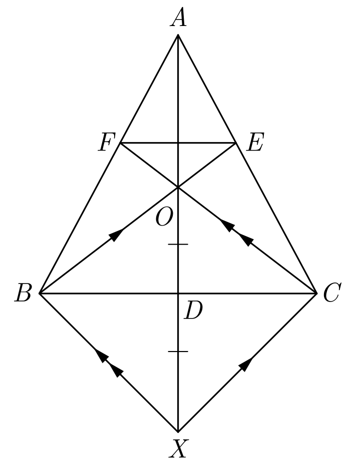
or, $$x = \frac{hd}{2h+d}$$

106. In ΔABC, AD is a median and O is any point on AD. BO and CO on producing meet AC and AB at E and F respectively. Now AD is produced to X such that $OD = DX$ as shown in figure.

Prove that :

(1) $EF || BC$

(2) $AO : AX = AF : AB$



Ans : [Board Term-1 2015]

Since BC and OX bisect each other, $BXCO$ is a parallelogram. Therefore $BE || XC$ and $BX || CF$.

In ΔABX, by BPT we get,

$$\frac{AF}{FB} = \frac{AO}{OX} \quad ..(1)$$

In ΔAXC, $$\frac{AE}{EC} = \frac{AO}{OX} \quad ...(2)$$

From (1) and (2) we get

$$\frac{AF}{FB} = \frac{AE}{EC}$$

By converse of BPT we have

$$EF || BC$$

From (1) we get $$\frac{OX}{OA} = \frac{FB}{AF}$$

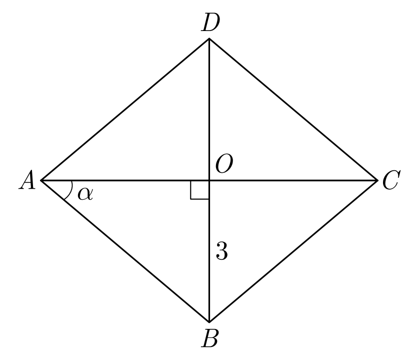
$$\frac{OX+OA}{OA} = \frac{FB+AF}{AF}$$

$$\frac{AX}{OA} = \frac{AB}{AF}$$

$$\frac{AO}{AX} = \frac{AF}{AB}$$

Thus $AO : AX = AF : AB$ Hence Proved

107. $ABCD$ is a rhombus whose diagonal AC makes an angle α with AB. If $\cos\alpha = \frac{2}{3}$ and $OB = 3$ cm, find the length of its diagonals AC and BD.



Ans : [Board Term-1 2013]

We have $\cos\alpha = \frac{2}{3}$ and $OB = 3$ cm

In ΔAOB, $\cos\alpha = \frac{2}{3} = \frac{AO}{AB}$

Let $OA = 2x$ then $AB = 3x$

Now in right angled triangle ΔAOB we have

$$AB^2 = AO^2 + OB^2$$

$$(3x)^2 = (2x)^2 + (3)^2$$

$$9x^2 = 4x^2 + 9$$

$$5x^2 = 9$$

$$x = \sqrt{\frac{9}{5}} = \frac{3}{\sqrt{5}}$$

Hence, $$OA = 2x = 2\left(\frac{3}{\sqrt{5}}\right) = \frac{6}{\sqrt{5}} \text{ cm}$$

and $$AB = 3x = 3\left(\frac{3}{\sqrt{5}}\right) = \frac{9}{\sqrt{5}} \text{ cm}$$

Diagonal $$BD = 2 \times OB = 2 \times 3 = 6 \text{ cm}$$

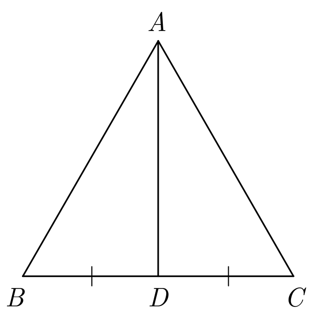
and $$AC = 2AO$$

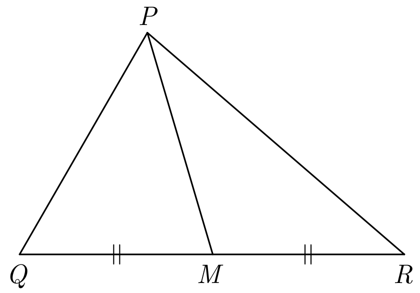
$$= 2 \times \frac{6}{\sqrt{5}} = \frac{12}{\sqrt{5}} \text{ cm}$$

108. In ΔABC, AD is the median to BC and in ΔPQR, PM is the median to QR. If $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$. Prove that $\Delta ABC \sim \Delta PQR$.

Ans : [Board Term-1 2012, 2013]

As per given condition we have drawn the figure below.





In ΔABC AD is the median, therefore

$$BC = 2BD$$

and in ΔPQR, PM is the median,

$$QR = 2QM$$

Given, $$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BC}{QR}$$

or, $$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{2BD}{2QM}$$

In triangles ABD and PQM,

$$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{BD}{QM}$$

By SSS similarity we have

$$\Delta ABD \sim \Delta PQM$$

By CPST we have

$$\angle B = \angle Q,$$

In ΔABC and ΔPQR,

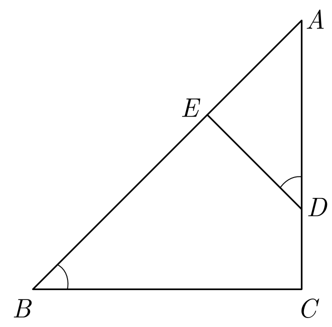
$$\frac{AB}{PQ} = \frac{BC}{QR}$$

By SAS similarity we have

$$\angle B = \angle Q,$$

Thus $\Delta ABC \sim \Delta PQR$. Hence Proved.

109. In ΔABC, if $\angle ADE = \angle B$, then prove that $\Delta ADE \sim \Delta ABC$.
Also, if $AD = 7.6$ cm, $AE = 7.2$ cm, $BE = 4.2$ cm and $BC = 8.4$ cm, then find DE.



Ans : [Board Term-1 2015]

In ΔADE and ΔABC, $\angle A$ is common.

and we have $\angle ADE = \angle ABC$

Due to AA similarity,

$$\Delta ADE \sim \Delta ABC$$

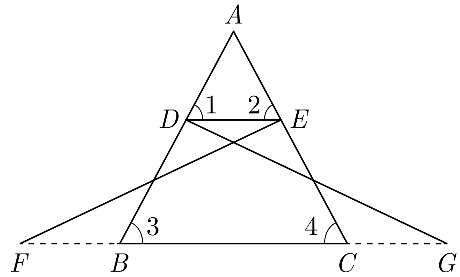
$$\frac{AD}{AB} = \frac{DE}{BC}$$

$$\frac{AD}{AE + BE} = \frac{DE}{BC}$$

$$\frac{7.6}{4.2 + 4.2} = \frac{DE}{8.4}$$

$$DE = \frac{7.6 \times 8.4}{11.4} = 5.6 \text{ cm}$$

110. In the following figure, $\Delta FEC \cong \Delta GBD$ and $\angle 1 = \angle 2$. Prove that $\Delta ADE \cong \Delta ABC$.



Ans : [Board Term-1 2012]

Since $\Delta\,FEC \cong \Delta\,GBD$

$$EC = BD \qquad ...(1)$$

Since $\angle 1 = \angle 2$, using isosceles triangle property

$$AE = AD \qquad ...(2)$$

From equation (1) and (2), we have

$$\frac{AE}{EC} = \frac{AD}{BD}$$

$$DE \parallel BC, \qquad \text{(Converse of BPT)}$$

Due to corresponding angles we have

$$\angle 1 = \angle 3 \text{ and } \angle 2 =$$

Thus in $\Delta\,ADE$ and $\Delta\,ABC$,

$$\angle A = \angle A$$

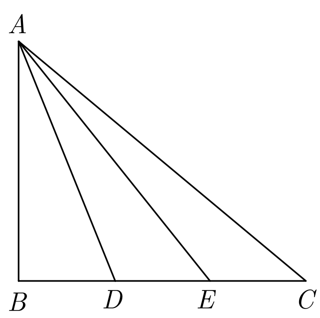
$$\angle 1 = \angle 3$$

$$\angle 2 = \angle 4$$

Sy by AAA criterion of similarity,

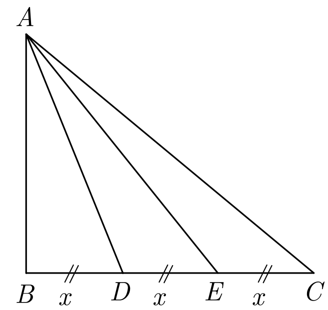
$$\Delta\,ADE \sim \Delta\,ABC \qquad \text{Hence proved}$$

111. In the given figure, D and E trisect BC. Prove that $8AE^2 = 3AC^2 + 5AD^2$.



Ans : [Board Term-1 2013]

As per given condition we have drawn the figure below.



Since D and E trisect BC, let $BD = DE = EC$ be x.

Then $\qquad BE = 2x$ and $BC = 3x$

In $\Delta\,ABE$, $\qquad AE^2 = AB^2 + BE^2 = AB^2 + 4x^2 \quad ...(1)$

In $\Delta\,ABC$, $\qquad AC^2 = AB^2 + BC^2 = AB^2 + 9x^2 \quad ...(2)$

In $\Delta\,ADB$, $\qquad AD^2 = AB^2 + BD^2 = AB^2 + x^2 \quad ...(3)$

Multiplying (2) by 3 and (3) by 5 and adding we have

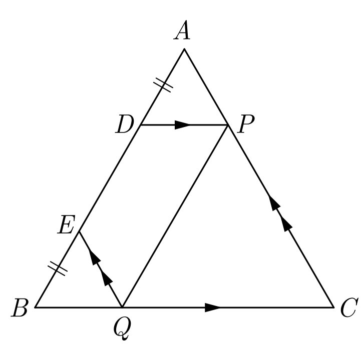
$$\begin{aligned} 3AC^2 + 5AD^2 &= 3(AB^2 + 9x^2) + (AB^2 + x^2) \\ &= 3AB^2 + 27x^2 + 5AB^2 + 5x^2 \\ &= 8AB^2 + 32x^2 \\ &= 8(AB^2 + 4x^2) = 8AE^2 \end{aligned}$$

Thus $\quad 3AC^2 + 5AD^2 = 8AE^2 \qquad$ Hence Proved

112. Let ABC be a triangle D and E be two points on side AB such that $AD = BE$. If $DP \parallel BC$ and $EQ \parallel AC$, then prove that $PQ \parallel AB$.

Ans : [Board Term-1 2012]

As per given condition we have drawn the figure below.



In $\Delta\,ABC$, $\qquad DP \parallel BC$

By BPT we have $\qquad \frac{AD}{DB} = \frac{AP}{PC}, \qquad ...(1)$

Similarly, in $\Delta\,ABC$, $\qquad EQ \parallel AC$

$$\frac{BQ}{QC} = \frac{BE}{EA} \quad ...(2)$$

From figure, $EA = AD + DE$

$= BE + ED \quad (BE = AD)$

$= BD$

Therefore equation (2) becomes,

$$\frac{BQ}{QC} = \frac{AD}{BD} \quad ...(3)$$

From (1) and (3), we have

$$\frac{AP}{PC} = \frac{BQ}{QC}$$

By converse of BPT,

$PQ \parallel AB$ Hence Proved

113. Prove that in a right triangle, the square of the hypotenuse is equal to sum of squares of other two sides. [Board 2020 Delhi Basic, 2019 Delhi, 2018]

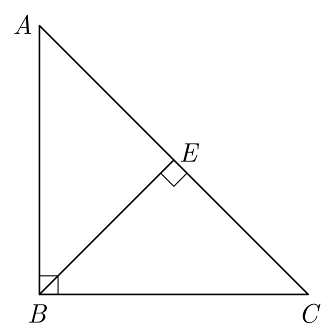
or

Prove that in a right triangle, the square of the hypotenuse is equal to sum of squares of other two sides. Using the above result, prove that, in rhombus $ABCD$, $4AB^2 = AC^2 + BD^2$.

Ans : [Board Term -2 SQP 2017, 2015]

(1) As per given condition we have drawn the figure below. Here $AB \perp BC$.

We have drawn $BE \perp AC$



In ΔAEB and ΔABC $\angle A$ common and

$\angle E = \angle B$ (each 90°)

By AA similarity we have

$$\Delta AEB \sim \Delta ABC$$

$$\frac{AE}{AB} = \frac{AB}{AC}$$

$$AB^2 = AE \times AC$$

Now, in ΔCEB and ΔCBA, $\angle C$ is common and

$\angle E = \angle B$ (each 90°)

By AA similarity we have

$$\Delta AEB \sim \Delta CBA$$

$$\frac{CE}{BC} = \frac{BC}{AC}$$

$$BC^2 = CE \times AC \quad ...(2)$$

Adding equation (1) and (2) we have

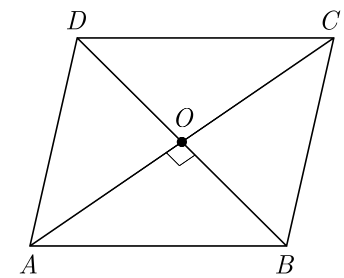
$$AB^2 + BC^2 = AE \times AC + CE \times AC$$

$$= AC(AE + CE)$$

$$= AC \times AC$$

Thus $AB^2 + BC^2 = AC^2$ Hence proved

(2) As per given condition we have drawn the figure below. Here $ABCD$ is a rhombus.



We have drawn diagonal AC and BD.

$$AO = OC = \frac{1}{2}AC$$

and $$BO = OD = \frac{1}{2}BD$$

$$AC \perp BD$$

Since diagonal of rhombus bisect each other at right angle,

$$\angle AOB = 90^\circ$$

$$AB^2 = OA^2 + OB^2$$

$$= \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2$$

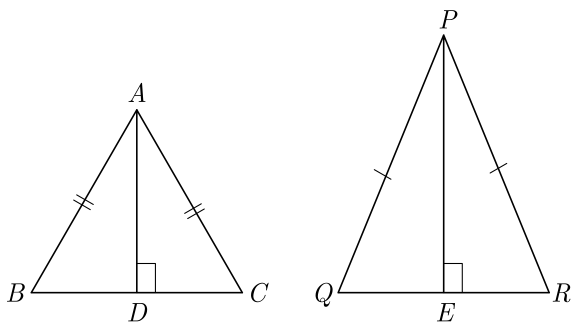
$$= \frac{AC^2}{4} + \frac{BD^2}{4}$$

or $4AB^2 = AC^2 + BD^2$ Hence proved

114. Vertical angles of two isosceles triangles are equal. If their areas are in the ratio $16:25$, then find the ratio of their altitudes drawn from vertex to the opposite side.

Ans : [Board Term-1 2015]

As per given condition we have drawn the figure below.



Here $\angle A = \angle P \ \angle B = \angle C$ and $\angle Q = \angle R$

Let $\angle A = \angle P$ be x.

In ΔABC, $\angle A + \angle B + \angle C = 180^{\circ}$

$$x + \angle B + \angle B = 180^{\circ} \qquad (\angle B = \angle C)$$

$$2\angle B = 180^{\circ} - x$$

$$\angle B = \frac{180^{\circ} - x}{2} \qquad ...(1)$$

Now, in ΔPQR,

$$\angle P + \angle Q + \angle R = 180^{\circ} \qquad (\angle Q = \angle R)$$

$$x^2 + \angle Q + \angle Q = 180^{\circ}$$

$$2\angle Q = 180^{\circ} - x$$

$$\angle Q = \frac{180^{\circ} - x}{2}$$

In ΔABC and ΔPQR,

$$\angle A = \angle P \qquad \text{[Given]}$$

$$\angle B = \angle Q \qquad \text{[From eq. (1) and (2)]}$$

Due to AA similarity,

$$\Delta ABC \sim \Delta PQR$$

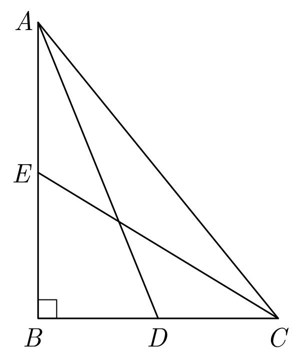
Now $$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PE^2}$$

$$\frac{16}{25} = \frac{AD^2}{PE^2}$$

$$\frac{4}{5} = \frac{AD}{PE}$$

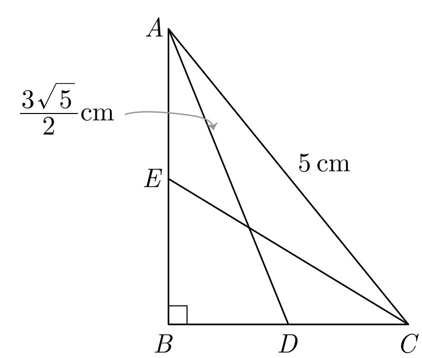
Thus $$\frac{AD}{PE} = \frac{4}{5}$$

115. In the figure, ABC is a right triangle, right angled at B. AD and CE are two medians drawn from A and C respectively. If $AC = 5$ cm and $AD = \frac{3\sqrt{5}}{2}$ cm, find the length of CE.



Ans : [Board Term-1 2013]

We have redrawn the given figure as below.



Here in ΔABC, $\angle B = 90^{\circ}$, AD and CE are two medians.

Also we have $AC = 5$ cm and $AD = \frac{3\sqrt{5}}{2}$.

By Pythagoras theorem we get

$$AC^2 = AB^2 + BC^2 = (5)^2 = 25 \qquad ...(1)$$

In ΔABD, $$AD^2 = AB^2 + BD^2$$

$$\left(\frac{3\sqrt{5}}{2}\right)^2 = AB^2 + \frac{BC^2}{4}$$

$$\frac{45}{4} = AB^2 + \frac{BC^2}{4} \qquad ...(2)$$

In ΔEBC, $$CE^2 = BC^2 + \frac{AB^2}{4} \qquad ..(3)$$

Subtracting equation (2) from equation (1),

$$\frac{3BC^2}{4} = 25 - \frac{45}{4} = \frac{55}{4}$$

$$BC^2 = \frac{55}{3} \quad ...(4)$$

From equation (2) we have

$$AB^2 + \frac{55}{12} = \frac{45}{4}$$

$$AB^2 = \frac{45}{4} - \frac{55}{12} = \frac{20}{3}$$

From equation (3) we get

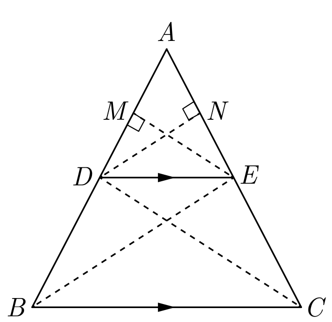
$$CE^2 = \frac{55}{3} + \frac{20}{3 \times 4} = \frac{240}{12} = 20$$

Thus $CE = \sqrt{20} = 2\sqrt{5}$ cm.

116. If a line drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. Prove it.

Ans : [Board 2019 OD, SQP 2020 STD, 2012]

A triangle ABC is given in which $DE || BC$. We have drawn $DN \perp AE$ and $EM \perp AD$ as shown below. We have joined BE and CD.



In ΔADE,

$$\text{area } (\Delta ADE) = \frac{1}{2} \times AE \times DN \quad ...(1)$$

In ΔDEC,

$$\text{area } (\Delta DCE) = \frac{1}{2} \times CE \times DN \quad ...(2)$$

Dividing equation (1) by (2) we have,

$$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEC)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times CE \times DN}$$

or, $$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEC)} = \frac{AE}{CE} \quad ...(3)$$

Now in ΔADE,

$$\text{area}(\Delta ADE) = \frac{1}{2} \times AD \times EM \quad ...(4)$$

and in ΔDEB,

$$\text{area}(\Delta DEB) = \frac{1}{2} \times EM \times BD \quad ...(5)$$

Dividing eqn. (4) by eqn. (5),

$$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEB)} = \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times BD \times EM}$$

or, $$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEB)} = \frac{AD}{BD} \quad ...(6)$$

Since ΔDEB and ΔDEC lie on the same base DE and between two parallel lines DE and BC.

$$\text{area}(\Delta DEB) = \text{area}(\Delta DEC)$$

From equation (3) we have

$$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEB)} = \frac{AE}{CE} \quad ...(7)$$

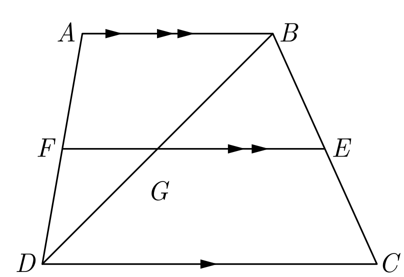
From equations (6) and (7) we get

$$\frac{AE}{CE} = \frac{AD}{BD}.$$ Hence proved.

117. In a trapezium $ABCD, AB || DC$ and $DC = 2AB$. $EF = AB$, where E and F lies on BC and AD respectively such that $\frac{BE}{EC} = \frac{4}{3}$ diagonal DB intersects EF at G. Prove that, $7EF = 11AB$.

Ans : [Board Term-1 2012]

As per given condition we have drawn the figure below.



In trapezium $ABCD$,

$$AB || DC \text{ and } DC = 2AB.$$

Also, $$\frac{BE}{EC} = \frac{4}{3}$$

Thus $$EF || AB || CD$$

$$\frac{AF}{FD} = \frac{BE}{EC} = \frac{4}{3}$$

In ΔBGE and ΔBDC, $\angle B$ is common and due to corresponding angles,

$$\angle BEG = \angle BCD$$

Due to AA similarity we get

$$\Delta BGE \sim \Delta BDC$$

, $$\frac{EG}{CD} = \frac{BE}{BC} \qquad \text{...(1)}$$

As, $$\frac{BE}{EC} = \frac{4}{3}$$

$$\frac{BE}{BE+EC} = \frac{4}{4+3} = \frac{4}{7}$$

$$\frac{BE}{BC} = \frac{4}{7} \qquad \text{...(2)}$$

From (1) and (2) we have

$$\frac{EG}{CD} = \frac{4}{7}$$

, $$EG = \frac{4}{7}CD \qquad \text{...(3)}$$

Similarly, $$\Delta DGF \sim \Delta DBA$$

$$\frac{DF}{DA} = \frac{FG}{AB}$$

$$\frac{FG}{AB} = \frac{3}{7}$$

$$FG = \frac{3}{7}AB \qquad \text{...(4)}$$

$$\left[\frac{AF}{AD} = \frac{4}{7} = \frac{BE}{BC} \Rightarrow \frac{EC}{BC} = \frac{3}{7} = \frac{DE}{DA}\right]$$

Adding equation (3) and (4) we have

$$EG + FG = \frac{4}{7}DC + \frac{3}{7}AB$$

$$EF = \frac{4}{7} \times (2AB) + \frac{3}{7}AB$$

$$= \frac{8}{7}AB + \frac{3}{7}AB = \frac{11}{7}AB$$

$$7EF = 11AB$$ Hence proved.

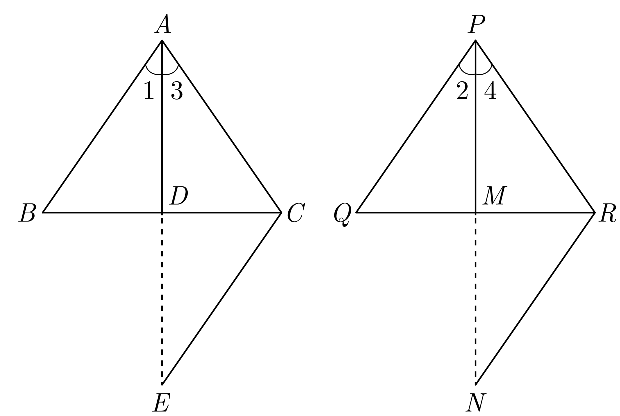
118. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$.

Ans : [Board Term-1 2012]

It is given that in ΔABC and ΔPQR, AD and PM are their medians,

such that $$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$$

We have produce AD to E such that $AD = DE$ and produce PM to N such that $PM = MN$. We join CE and RN. As per given condition we have drawn the figure below.



In ΔABD and ΔEDC,

$$AD = DE \qquad \text{(By construction)}$$

$$\angle ADB = \angle EDC \qquad \text{(VOA)}$$

$$BD = DC \qquad (AD \text{ is a median})$$

By SAS congruency

$$\Delta ABD \cong \Delta EDC$$

$$AB = CE \qquad \text{(By CPCT)}$$

Similarly, $$PQ = RN \text{ and } \angle A = \angle 2$$

$$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR} \qquad \text{(Given)}$$

, $$\frac{CE}{RN} = \frac{2AD}{2PM} = \frac{AC}{PR}$$

$$\frac{CE}{RN} = \frac{AE}{PN} = \frac{AC}{PR}$$

By SSS similarity, we have

$$\Delta AEC \sim \Delta PNR$$

$$\angle 3 = \angle 4$$

$$\angle 1 = \angle 2$$

$$\angle 1 + \angle 3 = \angle 2 + \angle 4$$

By SAS similarity, we have

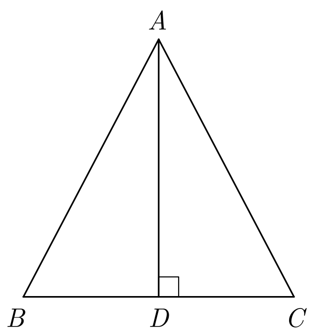
$$\Delta ABC \sim \Delta PQR$$ Hence Proved

119. In $\Delta ABC, AD \perp BC$ and point D lies on BC such that $2DB = 3CD$. Prove that $5AB^2 = 5AC^2 + BC^2$.

Ans : [Board Term-1 2015]

It is given in a triangle $\Delta ABC, AD \perp BC$ and point D lies on BC such that $2DB = 3CD$.

As per given condition we have drawn the figure below.



Since $2DB = 3CD$

$$\frac{DB}{CD} = \frac{3}{2}$$

Let DB be $3x$, then CD will be $2x$ so $BC = 5x$.

Since $\angle D = 90^\circ$ in ΔADB, we have

$$AB^2 = AD^2 + DB^2 = AD^2 + (3x)^2$$
$$= AD^2 + 9x^2$$
$$5AB^2 = 5AD^2 + 45x^2$$
$$5AD^2 = 5AB^2 - 45x^2 \quad ...(1)$$

and $$AC^2 = AD^2 + CD^2 = AD^2 + (2x)^2$$
$$= AD^2 + 4x^2$$
$$5AC^2 = 5AD^2 + 20x^2$$
$$5AD^2 = 5AC^2 - 20x^2 \quad ...(2)$$

Comparing equation (1) and (2) we have

$$5AB^2 - 45x^2 = 5AC^2 - 20x^2$$
$$5AB^2 = 5AC^2 - 20x^2 + 45x^2$$
$$= 5AC^2 + 25x^2$$
$$= 5AC^2 + (5x)^2$$
$$= 5AC^2 + BC^2 \quad [BC = 5x]$$

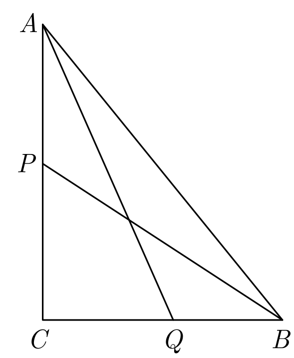
Therefore $5AB^2 = 5AC^2 + BC^2$ Hence proved

120. In a right triangle ABC, right angled at C. P and Q are points of the sides CA and CB respectively, which divide these sides in the ratio $2:1$.

Prove that : $$9AQ^2 = 9AC^2 + 4BC^2$$
$$9BP^2 = 9BC^2 + 4AC^2$$
$$9(AQ^2 + BP^2) = 13AB^2$$

Ans :

As per given condition we have drawn the figure below.



Since P divides AC in the ratio $2:1$

$$CP = \frac{2}{3}AC$$

and Q divides CB in the ratio $2:1$

$$QC = \frac{2}{3}BC$$

$$AQ^2 = QC^2 + AC^2$$
$$= \frac{4}{9}BC^2 + AC^2$$

or, $$9AQ^2 = 4BC^2 + 9AC^2 \quad ...(1)$$

Similarly, we get

$$9BP^2 = 9BC^2 + 4AC^2 \quad ...(2)$$

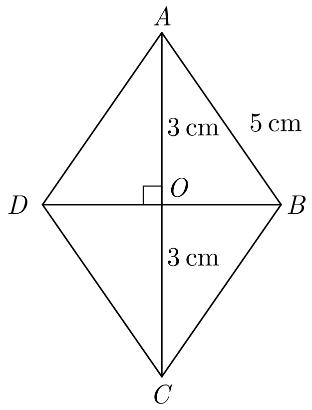
Adding equation (1) and (2), we get

$$9(AQ^2 + BP^2) = 13AB^2$$

121. Find the length of the second diagonal of a rhombus, whose side is 5 cm and one of the diagonals is 6 cm.

Ans :

As per given condition we have drawn the figure below.



We have $AB = BC = CD = AD = 5$ cm and $AC = 6$ cm

Since $AO = OC$, $\qquad AO = 3$ cm

Here ΔAOB is right angled triangle as diagonals of rhombus intersect at right angle.

By Pythagoras theorem,

$$OB = 4 \text{ cm.}$$

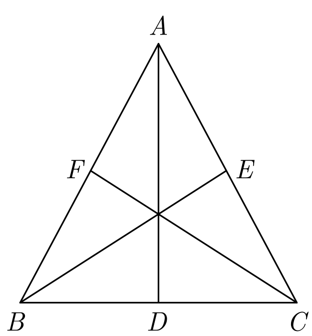
Since $DO = OB, BD = 8$ cm, length of the other diagonal $= 2(BO)$ where $BO = 4$ cm

Hence $\qquad BD = 2 \times BO = 2 \times 4 = 8$ cm

122. Prove that three times the sum of the squares of the sides of a triangle is equal to four times the sum of the squares of the medians of the triangle.

Ans :

As per given condition we have drawn the figure below.



In triangle sum of squares of any two sides is equal to twice the square of half of the third side, together with twice the square of median bisecting it.

If AD is the median,

$$AB^2 + AC^2 = 2\left\{AD^2 + \frac{BC^2}{4}\right\}$$

$$2(AB^2 + AC^2) = 4AD^2 + BC^2 \qquad ...(1)$$

Similarly by taking BE and CF as medians,

$$2(AB^2 + BC^2) = 4BE^2 + AC^2 \qquad ...(2)$$

and $$2(AC^2 + BC^2) = 4CF^2 + AB^2 \qquad ...(3)$$

Adding, (1), (2) and (iii), we get

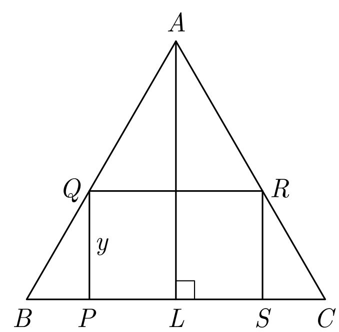
$$3(AB^2 + BC^2 + AC^2) = 4(AD^2 + BE^2 + CF^2)$$

Hence proved

123. ABC is an isosceles triangle in which $AB = AC = 10$ cm $BC = 12$ cm $PQRS$ is a rectangle inside the isosceles triangle. Given $PQ = SR = y$, $PS = PR = 2x$. Prove that $x = 6 - \frac{3y}{4}$.

Ans :

As per given condition we have drawn the figure below.



Here we have drawn $AL \perp BC$.

Since it is isosceles triangle, AL is median of BC,

$$BL = LC = 6 \text{ cm.}$$

In right ΔALB, by Pythagoras theorem,

$$AL^2 = AB^2 - BL^2$$

$$= 10^2 - 6^2 = 64 = 8^2$$

Thus $AL = 8$ cm.

In ΔBPQ and ΔBLA, angle $\angle B$ is common and

$$\angle BPQ = \angle BLA = 90^\circ$$

Thus by AA similarity we get

$$\Delta BPQ \sim \angle BLA$$

$$\frac{PB}{PQ} = \frac{BL}{AL}$$

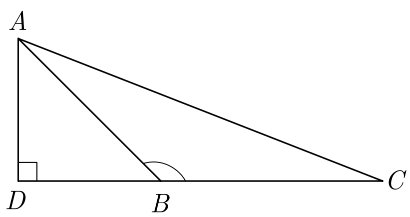
$$\frac{6-x}{y} = \frac{6}{8}$$

$$x = 6 - \frac{3y}{4} \qquad \text{Hence proved.}$$

124. If ΔABC is an obtuse angled triangle, obtuse angled at B and if $AD \perp CB$. Prove that :
$AC^2 = AB^2 + BC^2 + 2BC \times BD$

Ans : [Board 2020 Delhi Basic]

As per given condition we have drawn the figure below.



In ΔADB, by Pythagoras theorem

$$AB^2 = AD^2 + BD^2 \qquad ...(1)$$

In ΔADC, By Pythagoras theorem,

$$AC^2 = AD^2 + CD^2$$
$$= AD^2 + (BC + BD)^2$$
$$= AD^2 + BC^2 + 2BC \times BD + BD^2$$
$$= (AD^2 + BD^2) + 2BC \times BD$$

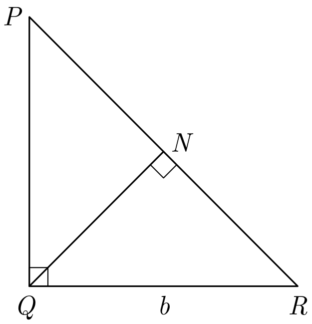
Substituting $(AD^2 + BD^2) = AB^2$ we have

$$AC^2 = AB^2 + BC^2 + 2BC \times BD$$

125. If A be the area of a right triangle and b be one of the sides containing the right angle, prove that the length of the altitude on the hypotenuse is $\frac{2Ab}{\sqrt{b^4 + 4A^2}}$.

Ans :

As per given condition we have drawn the figure below.



Let $QR = b$, then we have

$$A = ar(\Delta PQR)$$
$$= \frac{1}{2} \times b \times PQ$$
$$PQ = \frac{2 \cdot A}{b} \qquad ...(1)$$

Due to AA similarity we have

$$\Delta PNQ \sim \Delta PQR$$
$$\frac{PQ}{PR} = \frac{NQ}{QR} \qquad ...(2)$$

From ΔPQR

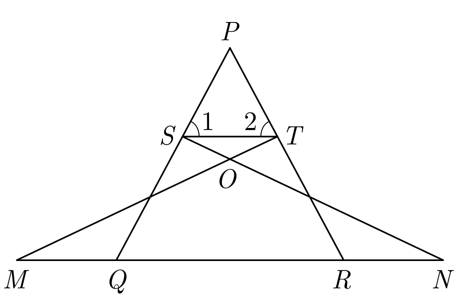
$$PQ^2 + QR^2 = PR^2$$
$$\frac{4A^2}{b^2} + b^2 = PR^2$$
$$PR = \sqrt{\frac{4A^2 + b^4}{b^2}}$$

Equation (2) becomes

$$\frac{2A}{b \times PR} = \frac{NQ}{b}$$
$$NQ = \frac{2A}{PR}$$

Altitude, $$NQ = \frac{2Ab}{\sqrt{4A^2 + b^4}} \qquad \text{Hence Proved.}$$

126. In given figure $\angle 1 = \angle 2$ and $\Delta NSQ \sim \Delta MTR$, then prove that $\Delta PTS \sim \Delta PRO$.



Ans : [Board Term-1 SQP 2017]

We have $\Delta NSQ \cong \Delta MTR$

By CPCT we have

$$\angle SQN = \angle TRM$$

From angle sum property we get

$$\angle P + \angle 1 + \angle 2 = \angle P + \angle PQR + \angle PRQ$$

$$\angle 1 + \angle 2 = \angle PQR + \angle PRQ$$

Since $\angle 1 = \angle 2$ and $\angle PQR = \angle PRQ$ we get

$$2\angle 1 = 2\angle PQR$$

$$\angle 1 = \angle PQR$$

Also $\angle 2 = \angle QPR$ (common)

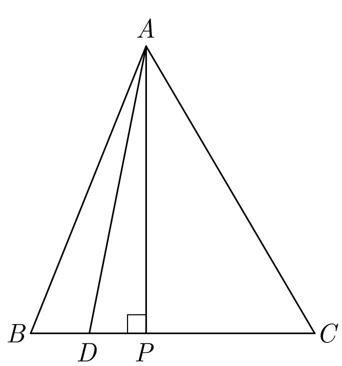
Thus by AAA similarity,

$$\Delta PTS \sim \Delta PRQ$$

127. In an equilateral triangle ABC, D is a point on the side BC such the $BD = \frac{1}{3}BC$. Prove that $9AD^2 = 7AB^2$.

Ans : [Board 2018, SQP 2017]

As per given condition we have shown the figure below. Here we have drawn $AP \perp BC$.



Here $AB = BC = CA$ and $BD = \frac{1}{3}BC$.

In ΔADP,

$$AD^2 = AP^2 + DP^2$$

$$= AP^2 + (BP - BD)^2$$

$$= AP^2 + BP^2 + BD^2 + 2BP \cdot BD$$

From ΔAPB using $AP^2 + BP^2 = AB^2$ we have

$$AD^2 = AB^2 + \left(\frac{1}{3}BC\right)^2 - 2\left(\frac{BC}{2}\right)\left(\frac{BC}{3}\right)$$

$$= AB^2 + \frac{AB^2}{9} - \frac{AB^2}{3} = \frac{7}{9}AB^2$$

$$9AD^2 = 7AB^2$$

Hence Proved